George Mason University

College of Education and Human Development

Elementary Education Program

ELED 552-003 – Mathematics Methods for the Elementary Classroom (Hybrid)

3 Credits, Fall 2022

Thursdays, 4:30 – 7:10 (Hybrid: Zoom & Thompson Hall 1020, Fairfax Campus)

Faculty

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Prerequisites/Corequisites

Admission to the elementary education licensure program

University Catalog Course Description

Introduces methods for teaching all children topics in arithmetic, geometry, algebra, probability, and statistics in elementary grades. Focuses on using manipulatives and technologies to explore mathematics and solve problems.

Course Overview

In this course, we will begin an inquiry into mathematics teaching and learning that will guide you in your first teaching job and give you the tools that will enable you to continue to inquire and learn as part of your work as a teacher. Class sessions will be interactive and will include a variety of hands-on experiences with concrete and virtual manipulatives appropriate for elementary school mathematics. We will explore the teaching of mathematics, investigating both *what* to teach and *how* to teach it. We will explore what it means to do mathematics and what it means to understand mathematics through individual, small group, and large group mathematical problem solving. We will investigate ways to represent understandings of mathematical concepts, communicate reasoning about mathematical ideas, and construct mathematical arguments. We will investigate and read

about ways children might represent mathematical concepts, looking at ways to help children build connections and see relationships among mathematical ideas. We will explore characteristics of a classroom environment conducive to mathematical learning by reading and discussing the importance of mathematical tasks, mathematical tools, the roles of teachers and students, and the assessment of mathematical understanding.

Doing Mathematics (Learning ***for*** practice): We will build our own *knowledge* of mathematics by closely investigating ideas in number sense and fraction concepts. Additionally, we will engage in several math tasks, extending our knowledge of mathematics and students' thinking, which will provide the foundation for your professional decision- making.

Examining Records of Practice (Learning ***from*** practice): Records of practice—such as videotapes of lessons taught by yourself or others, students' work, and teachers' professional writing—allow us to investigate the work of teaching mathematics and improve our own practice.

Trying Things Out (Learning ***in*** practice): Because teaching involves more than just having knowledge, we want you to engage in *enacting* the practices we are investigating as much as possible, so that you are developing the skills and professional decision-making that will make you a successful teacher.

Course Delivery Method

This course will be delivered face-to-face (~75%) and online (~25%) using a synchronous format via Blackboard Learning Management system (LMS) housed in the MyMason portal. You will log in to the Blackboard (Bb) course site using your Mason email name (everything before @masonlive.gmu.edu) and email password. The course site will be available on Aug 22, 2022. Individual session formats vary and may include small group/large group discussion, hands-on, interactive work, student presentations, and cooperative learning. Practical applications of theory are explored in group activities.

Under no circumstances, may candidates/students participate in online class sessions (either by phone or Internet) while operating motor vehicles. Further, as expected in a face-to-face class meeting, such online participation requires undivided attention to course content and communication.

Technical Requirements

To participate in this course, students will need to satisfy the following technical requirements:

High-speed Internet access with standard up-to-date browsers. To get a list of Blackboard's supported browsers see:
https://help.blackboard.com/Learn/Student/Getting_Started/Browser_Support#supported-browsers

To get a list of supported operating systems on different devices see:

https://help.blackboard.com/Learn/Student/Getting_Started/Browser_Support#tested-devices-and-operating-systems

- Students must maintain consistent and reliable access to their GMU email and Blackboard, as these are the official methods of communication for this course.
- Students will need a headset microphone for use with the Blackboard Collaborate web conferencing tool. [Delete this sentence if not applicable.]
- Students may be asked to create logins and passwords on supplemental websites and/or to download trial software to their computer or tablet as part of course requirements.
- The following software plug-ins for PCs and Macs, respectively, are available for free download:
    - Adobe Acrobat Reader: https://get.adobe.com/reader/
    - Windows Media Player: https://support.microsoft.com/en-us/help/14209/get-windows-media-player
    - Apple Quicktime Player: www.apple.com/quicktime/download/

Expectations

- Course Week: Our course week will begin on the day that our synchronous meetings take place as indicated on the Schedule of Classes.
- Log-in Frequency: Students must actively check the course Blackboard site and their GMU email for communications from the instructor, class discussions, and/or access to course materials at least 4 times per week. In addition, students must log-in for all scheduled online synchronous meetings.
- Participation: Students are expected to actively engage in all course activities throughout the semester, which includes viewing all course materials, completing course activities and assignments, and participating in course discussions and group interactions.
- Technical Competence: Students are expected to demonstrate competence in the use of all course technology. Students who are struggling with technical components of the course are expected to seek assistance from the instructor and/or College or University technical services.
- Technical Issues: Students should anticipate some technical difficulties during the semester and should, therefore, budget their time accordingly. Late work will not be accepted based on individual technical issues.
- Workload: Please be aware that this course is **not** self-paced. Students are expected to meet *specific deadlines* and *due dates* listed in the **Class Schedule** section of this syllabus. It is the student's responsibility to keep track of the weekly course schedule of topics, readings, activities and assignments due.
- Instructor Support: Students may schedule a one-on-one meeting to discuss course requirements, content or other course-related issues. Those unable to come to a Mason campus can meet with the instructor via telephone or web conference. Students should

email the instructor to schedule a one-on-one session, including their preferred meeting method and suggested dates/times.

- Netiquette: The course environment is a collaborative space. Experience shows that even an innocent remark typed in the online environment can be misconstrued. Students must always re-read their responses carefully before posting them, so others do not consider them as personal offenses. *Be positive in your approach with others and diplomatic in selecting your words*. Remember that you are not competing with classmates, but sharing information and learning from others. All faculty are similarly expected to be respectful in all communications.
- Accommodations: Online learners who require effective accommodations to insure accessibility must be registered with George Mason University Disability Services

Learner Outcomes or Objectives

This course is designed to enable students to do the following:

A. Know what constitutes the essential topics in mathematics of the modern early and intermediate grades school program.

B. Identify and use selected manipulatives and technology such as linking cubes, attribute blocks, geoboards, base-10 blocks, fraction circles, tangrams, calculators, and computers to teach appropriate mathematics content topics in the early and middle grades.

C. Identify and use various instructional strategies and techniques (cooperative and peer group learning, activity centers, laboratories and workshops, teacher-directed presentations, etc.) to teach mathematical content topics appropriate for the early and intermediate grades to all children, including those from non-mainstreamed populations.

D. Identify and use alternative methods for assessing students' work in mathematics in the early and intermediate grades.

E. Solve problems in the mathematical content areas of logic, number theory, geometry, algebra, probability, and statistics appropriate for adaptation to the early and intermediate grades.

F. Know and explain the learning progression in relation to the standards-based mathematics curriculum, the key elements of the National Council of Teachers of Mathematics Principles and Standards for School Mathematics, and the key elements of the Virginia Standards of Learning for Mathematics.

G. Understand the multiple representations of mathematical concepts and procedures.

H. Understand and use the five processes—reasoning mathematically, solving problems, communicating mathematics effectively, making mathematical connections, and using mathematical representations—at different levels of complexity

I. Explore the contributions of different cultures toward the development of mathematics, and the role of mathematics in culture and society.

J. Understand the relationship of math to science, the design process, and technology.

K. Understand, possess, and integrate the knowledge, skills, dispositions, and processes needed to support learners' achievement in an interdisciplinary manner in Virginia's Foundation Blocks for Early Learning: Comprehensive Standards for Four-Year-Olds and the Virginia Standards of Learning in English, mathematics, history and social science, science, and computer technology.

Additionally, this course supports the CEHD Core Values of collaboration, ethical leadership, research-based practice, social justice, and innovation. Statements of these goals are at http://cehd.gmu.edu/values/.

Professional Standards (Interstate Teacher Assessment and Support Consortium (InTASC))

Upon completion of this course, students will have met the following professional standards:

Course Student Outcomes (above)	**INTASC Standard (2013)**
A Essential math	#4
B Planning and Teaching using manipulatives	#7
C Instructional Strategies	#8
D Assessing	#6
E Problem Solving	#5
F Learner Development and understanding of Learning Progression	#2/#1

Course & PBA	INTASC
552 Math Student Assessment Interview	#4 Content Knowledge #1 & #2 Learner Development & Differences #6 Assessment

Required Texts

Van De Walle, J., Karp, K. S., & Bay-Williams, J. M. (2018). *Elementary and Middle School Mathematics: Teaching Developmentally.* (10th edition) New York: Pearson(2019:9780134802084)

Supply list will be shared on the first day of class

Other readings as assigned.

Course Performance Evaluation

Students are expected to submit all assignments on time in the manner outlined by the instructor (e.g., Blackboard)

- **Assignments and/or Examinations**

Assignments	Points
Math Autobiography, Surveys, Daily Participation, Digital Interactive Notebook (dINB) (30 points)	30 points
Math Lesson #1: Number Routine Rehearsal, Address the needs of multi-language learners (15 points)	15 points
Math Lesson #2: Group Problem Solving Lesson Rehearsal, Teaching with Technology (15 points)	15 points
Teach Math Lesson #1 or #2 in your placement (choice of one: number routine or problem solving lesson) **(5 points)**	5 points
Learning Trajectory Assessment Report: **Course Performance Based Assessment (35 points)** *(Due dates will be spread out throughout the semester)* LT Project Part 1. Description of the Learning Trajectory/Progression-Formative Assessment LT Project Part 2. Conduct Formative Assessment LT Project Part 3. Learning Goal and Instructional Activities LT Project Part 4. Reflection and Share out LT Project	35 points

Math Autobiography, Survey, Daily Participation, Digital Interactive Notebook (30 points)

Addresses Learner Outcomes: A, B, C, D, E, F

Reading and activity reflections will be done via Digital Interactive Notebook entries. First session, we will start with the Math Autobiography & Surveys and the last class we will have a final vision statement and post survey on Teaching practices. In class, students are expected to keep an digital interactive notebook where one can analyze and reflect on personal and group solution strategies, provide differentiated approaches to classroom activities, and actively participate in class discussions by applying field experiences and class readings.

It is expected that you attend all scheduled classes outlined within the syllabus. Absence from class to observe a religious holiday, to serve jury duty, or to participate in required military service, and

medical emergencies are exceptions to the above policy. If you need to be absent for any of these reasons, please make arrangements at least 48 hours in advance. In addition, you are expected to be on time to class each session unless 48 hours advance notice has been provided to the instructor.

This course operates with the assumption that knowledge is socially constructed and the most meaningful learning opportunities are those where you have the opportunity to offer and explore diverse perspectives with peers; therefore, you are expected to contribute to all class and online discussions and activities as well as genuinely listen to peers as they do the same.
You are expected to be prepared for each class, which means having completed all assigned readings and tasks for that class prior to the start of class.
Your participation includes completion of all synchronous and asynchronous application activities. This includes but is not limited to

- Contributions to whole group and small group discussions
- All digital notebook reflections and work

It is expected that you will not use cell phones (or computers) in class for purposes beyond enhancing your own learning of course content.

Evaluation
You will complete in-class and between class activities in your digital interactive notebook (dINB). Your notebook will be created for you and sent via email. Required weekly updates will be posted in daily slides, which can be found on Blackboard.***PLEASE NOTE*: If you are absent from class, you can earn half the daily points by completing all the between session classwork.

Daily Participation, Digital Interactive Notebook RUBRIC	Unsatisfactory (0 pts per day)	Proficient (2 pts per day)
Reading Reflection - includes work due prior to class session **Classwork (both individual and group)** - participation and collaboration during the session **Post-Class Reflection** - reflective and interactive work done after the class session. - Dialogue with the instructor	The student is absent from class and/or is not prepared for class (between class work is incomplete). Some or all work is missing.	The student is punctual and prepared for class (between class work is complete). The student actively participates and supports the members of the learning group and the members of the class. Work is completed in a distinguished way.

Math Lesson #1: Number Routine Rehearsal, Meeting the Needs of Multi-Language Learners (15 points)

Addresses Learner Outcomes: A, B, C, D, E, F

You are required to collaboratively plan a Math routine and rehearse it by teaching it to your classmates. After reflecting and revising with your group, you will then have the choice teach the routine in your field placement, and complete a reflection on the process (you may instead choose to teach the problem solving lesson). Each 10-15 minute Math Routine will include the six essential elements of a Math Routine, include planned WIDA adaptations for mult-language learners, and address an important concept. A Math Routine lesson plan template will be provided, to which you will add your anticipated student responses, your adaptations for multi-language learners, and expected series of questions.

Notes on planning for your group rehearsal for the Math Routine. Each group is expected to:

- prepare any materials needed for the Math Routine rehearsal and the adaptations for multi-language learners;
- anticipate possible student responses to the problems presented and plan your expected sequence of follow-up questions;
- Collect some of your classmates' work samples during the rehearsal (if created). Use these samples to debrief, reflect, and possibly revise. Prepare to share the work on slides.

Individually teach the routine to students in your placement. (Note: you will be required to teach only one lesson to students in your placement: a number routine or a problem-solving lesson). Collect samples of student work and reflect individually on student learning.

See rubric/Blackboard for more detail.

Part A: Collaboratively Plan & Rehearse Design slides to present the number routine using the six essential elements, and aligned to Virginia Standards of Learning and Common Core State Standards. Anticipate student responses using a learning progression. The plan should include Questions for Making Math Visible and encourage justification and equitable participation.	5 points
Part B: Address the needs of multi-language learners Use the WIDA framework to guide your decision-making as you adapt the number routine. Collect representative work samples for anticipated responses to later reflect on how the lesson supported math learning and equitable teaching practices for student learning.	5 points

Part C: Teach and Reflect: Rehearse in class. The group should debrief together in possible preparation for individually teachingthe number routine in a classroom of young students. However, each individual will provide an individual reflection slide. After rehearsing the routine, examine individual adult "student" work and reflect on how the lesson and the adaptations for multi-language learners supported math learning and equitable teaching practices. In this case, refer to the WIDA framework. If the routine did not support equitable teaching practices for language learners, explain why, and how you would change it. This reflection is an individual work.	5 points
Total points	15 points

Math Lesson #2: Group Problem Solving Lesson Rehearsal, Teaching with Technology (15 points)

Addresses Learner Outcomes: A, B, C, D, E, F

The lesson will be planned and taught by a small group and presented to classmates as a short, simulated lesson. Each group is expected to:

- teach a short problem-based, high cognitive demand task in class. This is a rehearsal.;
- use the EQTtech framework to evaluate and decide which manipulatives or representations are best suited for your students on this task;
- anticipate possible student responses by solving the problem using three, connected, representations (concrete, pictorial, abstract);
- prepare slides to share the task (notify the instructor the Sunday before you will teach it to the class);
- complete a written reflection on the analysis of student thinking after the rehearsal. Reflection will include how the candidate focused on promoting equitable participation and meaningful and rigorous learning for each and every child to promote digital equity.

Individually teach the problem solving lesson to students in your placement. (Note: you will be required to teach only one lesson to students in your placement: a number routine or a problem-solving lesson). Collect samples of student work and reflect individually on student learning.

See rubric/Blackboard for more detail.

Part A: Collaboratively Plan & Rehearse Design slides to present a problem-based, high cognitive demand task that is aligned to Virginia Standards of Learning and Common Core State Standards. Anticipate student responses using a learning progression. The plan should include a single task, a set of materials (based on the EQTtech framework—see Part B), and anticipated responses.	5 points
Part B: Make an equity-based choice of representations for teaching Use the EQTtech Framework to evaluate and select a digital or physical manipulative representation to provide for student use during the task. Describe how the tool you selected reflects equity-based access to the learning goal.	5 points
Part C: Teach and Reflect: Rehearse in class. The group should debrief together in possible preparation for individually teaching an abbreviated version of the problem-based task in a classroom of young students. However, each individual will provide an individual reflection slide. After rehearsing the problem-solving lesson, examine individual adult "student" work samples and reflect on how the lesson and the choices of technology supported math learning and equitable teaching practices. In this case, technology refers to both digital or hands-on manipulatives. If the lesson did not support equitable teaching practices, explain why, and how you would change it, including the choice of manipulatives or digital tools provided for students. This reflection is an individual work.	5 points
Total points	15 points

Teach and Reflect on Math Lesson #1 or #2 (Number Routine or Group Problem Solving Lesson Rehearsal) (5 points)

Addresses Learner Outcomes: A, B, C, D, E, F

After teaching a rehearsal of either the number routine or the problem solving lesson, debriefing and reflecting with your group members and individually, and then revising your lesson, teach one of the two lessons to students in your placement.

- Teach the lesson or routine, revised by your group after the rehearsal in class.
- Submit a reflection on what you learned from planning, rehearsing, debriefing, and teaching the revised lesson or routine. Submit to Blackboard.
- You are only required to teach **one** of the two lessons you will design.

Teach and Reflect: After rehearsing and enacting the routine in our class, examine and revise the plan as you reflect on how the lesson supported math learning and equitable teaching practices for student learning. If it did not, explain why, and how you are changing it to teach to your young students. Teach the lesson to your young students. Examine individual student work samples and include one or two in your final reflection on teaching the lesson. Reflect on what you learned from teaching this lesson to the students, and what you can take away for future teaching activities. This lesson reflection is an individual work.	5 points

Learning Trajectory Report : Formative Assessment with Sequenced Instructional Activities (with an individual or small group) Course Performance Based Assessment (40 points)
Addresses Learner Outcomes: A, B, C, D, F

In order to plan effective instruction, you will need to know how to assess children's knowledge of mathematical concepts. One way to assess children's thinking is a diagnostic assessment. This assignment has 4 parts:

Part 1. Description of the Learning Trajectory/Progression with visual of LT concept map. Choose a single formative assessment task that you will use with students. Explain how it relates to the learning trajectory. (~2 pages)

Part 2. Administer the formative assessment with a small group or individual. Share important screenshots of student work from the formative assessment (~3 pages)

Part 3. Future learning goal and instructional activities with screenshots of learning activity sequence and sample work (~ 1-2 pages with screen shots of activities)

Part 4. Reflection and share out (1 page) and be ready to share with a brief set of slides in class.

More details can be found in the assignment rubric found elsewhere on the syllabus and on Blackboard.

- **Other Requirements**
  - **Attendance:** It is your responsibility to attend all class sessions. You are held accountable for all information from each class session whether you are present or not. Reasons for any absence must be reported to the instructor in writing.
  - **Tardiness:** It is your responsibility to be on time for each class session. Reasons for any absence must be reported to the instructor in writing.

    Note: Faculty reserve the right to add, alter, or omit any assignment as necessary during the course of the semester. You will always receive advanced notice of any modifications.

- **Grading**

  Course Performance Evaluation Weighting

  The assignments across the semester are intended to further your understanding of what it means to teach, learn, and assess mathematics in light of current reforms in mathematics education. All assignments are to be turned in to your instructor on time.

  All assignments are to be completed by the date listed in the syllabus. Written work will not be accepted after the due date unless prior arrangements have been made with the instructor.

  Late work will not be accepted for full credit. If the student makes prior arrangements with the instructor, assignments turned in late will receive a 10% deduction from the grade per late day or any fraction thereof (including weekends and holidays).

  Given these policies, I also acknowledge these are unusual times and I care that you have opportunities to succeed. If something arises that threatens your ability to succeed in this course, communicate with me as soon as is reasonably possible.

Grading Policies

The mathematics education courses in GSE's Elementary Education Program integrate pedagogy and mathematics content appropriate for the elementary school grades. For students to earn a grade

of A in the course, they must demonstrate excellence in both the pedagogical knowledge and the content knowledge of mathematics appropriate at their level of teaching. Thus, the grading in the course is structured to help evaluate fair student excellence in both areas. Problem sets and assessment work focuses primarily on ascertaining student excellence in handling mathematics content appropriate for the elementary grades, and represents 50% of students' grades. Pedagogical knowledge is ascertained primarily from readings, assignments and participation in the course, and represents 50% of students' grades. Therefore, students who demonstrate excellence in both pedagogical knowledge and content knowledge receive grades of A.

At George Mason University course work is measured in terms of quantity and quality. A credit normally represents one hour per week of lecture or recitation or not fewer than two hours per week of laboratory work throughout a semester. The number of credits is a measure of quantity. The grade is a measure of quality. The university-wide system for grading graduate courses is as follows:

Grade	GRADING	Grade Points	Interpretation
A	94-100	4.00	Represents mastery of the subject through effort beyond basic requirements.
A-	90-93	3.67	
B+	85-89	3.33	Reflects an understanding of and the ability to apply theories and principles at a basic level
B	80-84	3.00	
C*	70-79	2.00	Denotes an unacceptable level of understanding and application of the basic elements of the course
F*	<69	0.00	

Note: "C" is not satisfactory for a licensure course.
"F" does not meet requirements of the Graduate School of Education

Professional Dispositions

Professional dispositions are to be displayed at all times while interacting with the instructor and other students. Cell phones are not to be used during class. Laptops are to be used for instructional purposes only.
See https://cehd.gmu.edu/students/polices-procedures/

Helpful Websites

Math Tasks/Resources/Manipulatives
University of Washington Number Talks website: https://tedd.org/
National Council of Teachers of Mathematics – Illuminations: https://illuminations.nctm.org/
Another Jo Boaler/Stanford University website w/K-12 math resources: https://www.youcubed.org/

Estimation 180 (Andrew Stadel's site – elementary and MS focused): https://estimation180.com/
Fawn Nguyen's website (MS Math focused): https://www.fawnnguyen.com/
Dan Meyer's website (more HS focused): https://blog.mrmeyer.com/ MS/HS 3-Act tasks
Graham Fletcher "3-Act" math lessons (ES focused): https://gfletchy.com/
Michael Pershan's blog, elementary school teacher: Math Mistakes blog

CCSS Helpful Websites:

CCSS Progressions Documents: http://ime.math.arizona.edu/progressions/
Lots of lesson plans and videos organized by Standard: insidemathematics.org
CCSS aligned tasks: illustrativemathematics.org
CCSS aligned tasks and assessments: map.mathshell.org/
Learning and Teaching with Learning Trajectories https://learningtrajectories.org/
Progressions Videos https://gfletchy.com/progression-videos/

Class Schedule

These topics and activities are subject to change to be responsive to the pace of the course. This is a projected sequence of due dates and planned readings. An "Always Current" schedule will be maintained in a Google Doc and will include updated information.

	Active learning & Discussion emphasis	**Readings due**	**Assignment due**
Aug 25, 2022 Session 1 **On Campus**	Welcome! What are Standards? Math Biography **Pedagogy:** Standards and **Teaching Practices** **Tools:** Virtual and Reality Manipulatives **Task:** Graphing to Know You! And What Does it Mean? Watch: Strength-Based Teaching and Learning in Mathematics Interview	Ch 1; pp. 4-9 Ch 2: pp. 19 – 28	

Sep 1, 2022 Session 2	Supporting Multi-Language Learners *Split It! Unpacking the Equipartitioning Learning Trajectory* article **Math Pedagogy:** Number Routines Equity-Based Practices **Task:** Introduce the dINB Memo and assignments and syllabus	Ch 6: pp 112-121 Article: *Split it! Unpacking the Equipartitioning Learning Trajectory*	**Math Biography due 9/5** **Vision Statement due 9/5** **Submit group's Number Routine choice by the end of class**
Sep 8, 2022 Session 3	Building Early Number Sense Early Childhood Counting	Ch 4: pp. 68 - 69 Ch 7: pp. 127 - 134, p. 150	
Sep 15, 2022 Session 4 **VIRTUAL Class**	Understanding our Base Ten Number System **Math Pedagogy:** High Cognitive Demand Tasks **Tools:** Base Ten Manipulatives **Task:** Packing Pencils	Ch 3: pp. 30-44 Ch 10: pp. 211-222, p. 236	**Number Routines Lesson Rehearsals, Group A**
Sep 22, 2022 Session 5	Designing Equitable Access to a Problem-Based Lesson **Pedagogy**: WIDA Framework **Task:** Caterpillar and Leaves **Math Pedagogy:** *Principles to Actions*- Teacher Practices and the Equity-Based Practices	Ch 4: pp. 55-66 Ch 3: pp. 48-52 Article: *Thinking through a Lesson Protocol*	**Number Routines Lesson Rehearsals, Groups B & C**
Sep 29, 2022 Session 6	Invented Strategies vs. Algorithms **(**Addition/Subtraction) **Task:** The Science Museum	Ch 11: pp. 240-260, pp. 270-271	**Submit group's Problem-Solving lesson choice by the next class**

	Math Pedagogy: TTLP lesson plan **Task:** The Science Museum **Tools:** Learning Trajectory Frameworks (OGAP Addition/Subtraction)	*Warning Signs! Avoid three common instructional moves that are generally followed by taking over children's thinking*	
Oct 6, 2022 Session 7 **VIRTUAL Class**	Invented Strategies vs. Algorithms Multiplication and Division **Math Pedagogy:** Respecting student thinking **Tools:** Learning Trajectory Frameworks (OGAP Multiplication and Division) **Math Pedagogy:** *Never Say Anything a Kid Can Say* **Discuss LT project**	Ch 12: pp. 274-276 and pp. 283-285 Article: *Never Say Anything a Kid Can Say*	**dINB and Reading Reflections** **#1 due Oct 9** *Respond back to instructor over the next two weeks.*
Oct 13, 2022 Session 8	Fraction Concepts **Math Pedagogy:** Learning Trajectories **Tools:** LT tables and fraction manipulatives **Task:** Pouring Paint and Unitizing Fractions with Cuisenaire Rods	Ch 14: Developing Fraction Concepts (part 1) pp. 338-358 Developing Fraction Concepts (part 2) pp. 359-371	**Problem Solving Lesson, Groups D, E** **Choose LT project topic**
Oct 20, 2022 Session 9 **VIRTUAL Class**	Making Meaning of Problem Situations **Math Pedagogy:** Introduction to 5 Practices for Orchestrating	Ch 8: pp. 154-164, pp. 166-173, pp. 180-181 Book chapter	**Problem Solving Lesson,** **Group F**

	Productive Mathematics Discussions **Tools:** Table of Problem Situations **Task:** Rugs or Bottle **CLT Time: LT project topic**		
Oct 27, 2022 Session 10	Making Sense of Fraction and Decimal Operations (addition and subtraction) **Math Pedagogy:** Interpreting and Choosing Manipulatives Peer comment/work time for LT Project **CLT time: Peer share**	Ch 15	**LT Project due date #1**
Nov 3, 2022 Session 11 **Asynchronous**	Fact Fluency **Math Pedagogy:** Four Dimensions of Fact Fluency **Tools:** Fact Fluency (*Fluency without Fear* Cards) and LTs for Fact Fluency **Task:** Alternatives to speed tests	Article: Kling and Bay-Williams (2014) *Assessing Basic Fact Fluency* Ch 9: pp. 184-189; pp. 205-210	*Asynchronous class. Conduct your LT formative assessment interviews*
Nov 10, 2022 Session 12	Making Sense of Fraction Operations (multiplication and division) **Math Pedagogy:** The development of multiplicative thinking and unit coordination **Task:** Sub Sharing task **CLT time: Peer review**	Ch 16	**LT Project due date #2** *This date is flexible because it requires cooperation from teachers and schools*

Nov 17, 2022 Session 13	Geometry is Classifying: Measuring is Comparing **Math Pedagogy:** Learning Trajectory **Tools:** Tools for any and every classroom **Task:** Exploration of Tools	Ch 18: pp. 461-468 (selection of pages 469 – 498) Ch 19: pp. 502 – 512 (selection of pages 515 – 541)	**dINB and Reading Reflections** **#2 due Nov 27** *Respond back to instructor over the next two weeks.*
Nov 24, 2022	**Thanksgiving Break** 	Ch 16	
Dec 1, 2022 Session 14 VIRTUAL Class	Algebraic Reasoning in the Early Grades **Math Pedagogy:** Generalizing **Tools:** variables and the equals sign	Ch 13: pp. 307-331	**LT Project due date #3.**
Dec 8, 2022 Session 15	Statistics and Data **Math Pedagogy:** What is typical? **Tools:** Daily Routines **Task:** Building towers Integer Operations **Math Pedagogy:** Inverse Operations	Ch 20: pp. 545 – 549 and pp. 568 – 577 Ch 22: pp. 614-624	**LT Project due date #4.** ***We will share in class.***

Note: Faculty reserves the right to alter the schedule as necessary, with notification to students.

Assignments	Points
Math Autobiography, Surveys, Daily Participation, Digital Interactive Notebook (dINB)	**30 points**
Math Lesson #1: Number Routine Rehearsal, Address the needs of multi-language learners	**15 points**
Math Lesson #2: Group Problem Solving Lesson Rehearsal, Teaching with Technology	**15 points**
Teach Math Lesson #1 or #2 in your placement (choice of one: number routine or problem solving lesson)	**5 points**
Learning Trajectory Assessment Report: Course Performance Based Assessment	**35 points**

Assessment Rubric(s)

Learning Trajectory Assessment Project

George Mason University College of Education and Human Development

Elementary Education Program

In the Elementary Education program, the **Learning Trajectory Assessment Project includes administering a formative assessment and designing bridging activities** during ELED 552 and is assessed by the instructor. The candidate must earn a score of 3 to be successful on this assignment. If a student does not earn a 3 on the assignment, they must meet with the course instructor or assessor prior to resubmitting. The data from this assessment are used to identify both best practice and identify gaps in developing and assessing a specific lesson plan to impact on individual learning.

STANDARDS

- **InTASC Standards**: 1, 2, 5, 6, 7, 8
- **CAEP Standards:** 1.1, 1.3, 1.4, 1.5
- **VDOE Standards:** 1, 2, 3, 4, 5

Standard #1: Learner Development. The teacher understands how learners grow and develop, recognizing that patterns of learning and development vary individually within and across the cognitive, linguistic, social, emotional, and physical areas, and designs and implements developmentally appropriate and challenging learning experiences.)

INTASC Standard #2: Learning Differences. The teacher uses understanding of individual differences and diverse cultures and communities to ensure inclusive learning environments that enable each learner to meet high standards)

Standard #4: Content Knowledge. The teacher understands the central concepts, tools of inquiry, and structures of the discipline(s) he or she teaches and creates learning experiences that make the discipline accessible and meaningful for learners to assure mastery of the content.

Standard #5: Application of Content. The teacher understands how to connect concepts and use differing perspectives to engage learners in critical thinking, creativity, and collaborative problem solving related to authentic local and global issues

InTASC Standard #6 Assessment. The teacher understands and uses multiple methods of assessment to engage learners in their own growth, to monitor learner progress, and to guide the teacher's and learner's decision making

Standard #7: Planning for Instruction. The teacher plans instruction that supports every student in meeting rigorous learning goals by drawing upon knowledge of content areas, curriculum, cross-disciplinary skills, and pedagogy, as well as knowledge of learners and the community context.

Standard #9: Professional Learning and Ethical Practice. The teacher engages in ongoing professional learning and uses evidence to continually evaluate his/her practice, particularly the effects of his/her choices and actions on others (learners, families, other professionals, and the community), and adapts practice to meet the needs of each learner.

THEMES

-  **Technology**
-  **Diversity**
-  **College & Career Ready**

ASSESSMENT OBJECTIVES

- The candidate will use knowledge of individual learning differences and assessment to develop an instructional plan for a learner with developmental, learning, physical or linguistic differences.
- The candidate will develop an assessment of learner progress.

RATIONALE

Lesson planning is an essential skill for an educator. A lesson plan is a road map for instruction. When planning teachers and teacher candidates need to answer four main questions:

- Who are my learners? (Context/Learner Needs)
- What do the learners need to know and be able to do? (Objectives/Goals)
- How will I get all learners to know and do the new tasks? (Teaching and learning strategies)
- How will I know the learning objectives were achieved? (Goals/Outcomes/Assessments)

The first step in planning is aligning the learning objectives with the goals/outcomes/assessments for the lesson. This should include considerations based on learner abilities, challenges, and prior knowledge. Before developing specific learning activities, determine how you will assess if learners have met the lesson objectives. Once you know how you will assess learning, you can develop activities that align instruction with the assessment. Additionally, a teacher must consider learner prior knowledge, how to differentiate to meet learner needs, and how to do so within the

time allotted. Lesson plans include pacing, transitions, checking for understanding, and ideas for re-teaching or extending learning based upon learner needs.

The planning process is the same whether you are planning a lesson for a class or for an individual. For this assessment you will develop an instructional plan for a learner with developmental, learning, physical or linguistic differences, including a plan for assessing the learner's progress.

ASSESSMENT DIRECTIONS

Candidates will develop an individualized plan for a child with a learning trajectory in mind. The individualized instruction and assessment plan should include the following sections:

Part 1. Description of the Learning Trajectory/Progression (~2 pages) **12 points**

Using your textbook, the learning progression document and other research around learning progression, describe the learning progression around your specific math concept. The candidate includes descriptions with specific examples from the research literature.

1a. Description of the learning trajectory/progression. Summarize the research around the learning progression around the topic chosen using your text. Create a concept map to illustrate the big ideas and scaffolding concepts.

1b. Description of common teaching and learning strategies. What are some of the common teaching strategies and the ways in which students develop the skills and understandings?

1c. Description of common student conceptions Specific references are made about common conceptions and misconceptions one might attend to, based on the research on the learner development using the text and progression document.

1d. Description of the formative assessment task Based on the LT research**,** select a Formative Assessment with multiple representations. Instructor must approve this choice. They can guide you to resources to help you. What task will students do that will allow you to assess their current knowledge along the learning trajectory?

Part 2. Administer a Quick Formative Assessment with a small group or individual (~ 2 – 3 pages) **12 points**

2a. Share any information **about the students** you are working with. What are the students' interests? Disposition towards mathematics? How they view math and how it's connected to their lives?

2b. Conduct the formative assessment with multiple representations. Use multiple and appropriate types representations, including pictorial, numeric, verbal and hands on manipulatives to assess student thinking

2c. Pose purposeful questions and look-fors to assess student's mathematical understanding Administer the assessment with prepared questions that gather information about student's interest, background and cultural information as well as specific questions linked to the tasks that probe thinking, and make math visible.

2d. Include a report of the assessment with strengths and the edges of their understanding with student work- What kinds of mathematics proficiency strengths do students in your class exhibit? What do the students look or sound like when they are exhibiting these strengths? What are the edges of their understanding and next steps for instructions?

Part 3. Learning Goal and Bridging Instructional Activities (1-2 pages) **8 points**

3a. Learning Goal (aka: target, benchmark, expectation). **Connection:** Now based on the formative assessment data, think about learning goals and instructional activities will you recommend

What should they learn? Identify learning goals within the learning progression and develop a rationale that supports why the objectives/goals are meaningful learning outcomes. (Virginia Standards of Learning (SOLs), College-and-Career-Ready skills, and other content specific objectives should be included in lesson plans.)

3b. Design and enact a sequence of bridging activities matched to the levels to advance the student's understanding. Describe at least three evidence-based instructional strategies (at least one with technology) that address the identified learning objectives/goals. Try one of the bridging activities and present how the activity advanced student thinking. Reflect on the learners' developmental skills using student work and verbal explanation..

3c. Include student work from the activities to describe where the student is in the developmental path and how you might advance them forward. Provide a rationale for instructional adaptations and accommodations needed. Describe the students' strength and areas for advancement.

Part 4. Reflection and Share out (~1 pages) **3 points**

Connection: Reflect on how part 1's LT research aligns with the overall assessment of the student understanding of this concept.

How did the assessment and did the bridging activities help you understand the where on the learning trajectory that one piece of student work falls?

What did you learn about learning trajectories? Reflect on your questioning skills? Did you plan for a variety of questions (i.e. gathering, probing, making math visible, reflecting & justifying questions)?What might you learn about how children learn mathematics from this assessment? How might a teacher use the diagnostic mathematics assessment to assess children?

Be ready to share with a brief set of slides

Resources

NCTM Process Standards and Virginia Standards of Learning (for students)

Mathematical Process Goals for Students
The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving
Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problem types. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication
Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students to clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

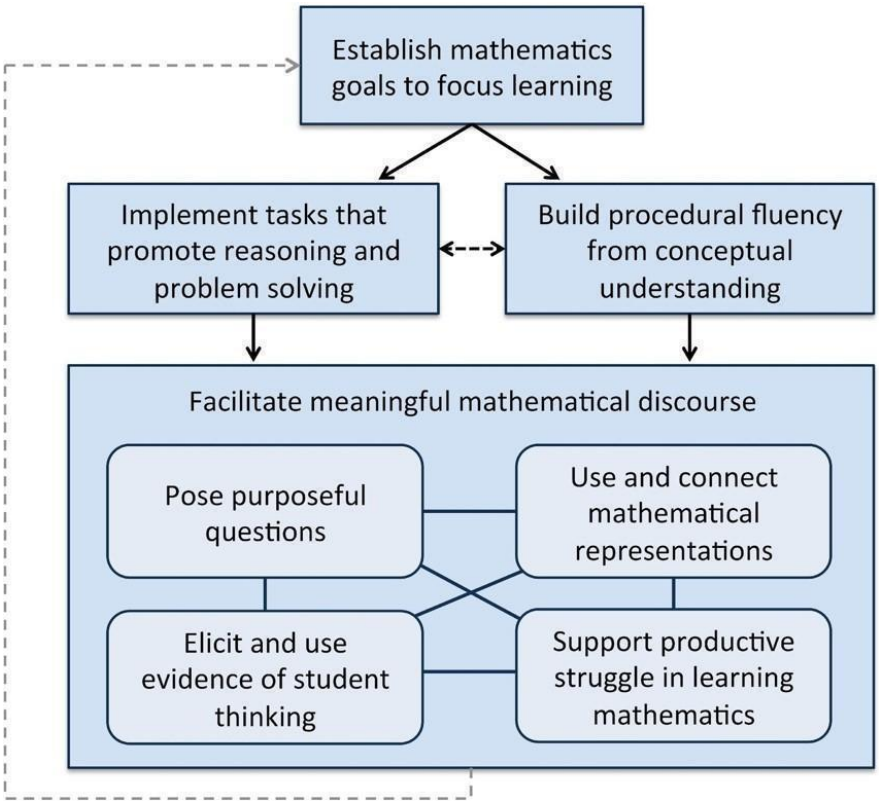
Mathematical Reasoning
Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections
Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Principles to Actions: 8 Teaching Practices (for teachers)



Mathematics Teaching Practices: Supporting Equitable Mathematics Teaching

Equity-Based Practices
Going deep with mathematics. Developing deep understanding of mathematics is a major goal of equity-based mathematics teaching practices (Aguirre 2009; Gutstein 2006). Lessons include high cognitive demand tasks that support and strengthen student development of the strands of mathematical proficiency, including conceptual understanding, procedural fluency, and problem solving and reasoning (National Governors Association Center for Best Practices and Council of Chief State School Officers 2010; National Research Council 2001a; Stein et al. 2000).
Leveraging multiple mathematical competencies.

Recognizing and positioning students' various mathematical backgrounds and competencies is a key equity-based practice (Featherstone et al. 2011; Horn 2012; Turner et al. 2012) All students have different mathematical strengths that can serve as resources for learning and teaching mathematics.
Affirming mathematics learners' identities. A positive, productive mathematics learner identity contributes to the mathematical learning of a child (Berry 2008; Boaler 2002; Martin 2000, 2009; Stinson 2008). Instruction that values multiple mathematical contributions, provides multiple entry points and promotes student participation in various ways (teams, groups, and so on) can aid the development of a student's mathematical learning identity.
Challenging spaces of marginality. Traditionally, mathematics learning has been an independent and isolating experience with a focus on lecture and seatwork. Further, students who do not perform well in this traditional classroom setting are often marginalized, ignored, or positioned as "dumb" (Boaler 2002; Jackson 2009). Practices that embrace student competencies, diminish status, and value multiple mathematical contributions as essential to cultivate (Aguirre et al. 2012; Featherstone et al. 2011; Horn 2012).
Drawing on multiple resources of knowledge. Equity-based teaching depends on the capacity to recognize and intentionally tap students' knowledge and experiences- mathematical, cultural, linguistic, peer, family, and community- as resources for mathematics teaching and learning. Drawing on this knowledge and experience includes helping students bridge everyday experiences to learn mathematics, capitalizing on linguistic resources to support mathematics learning, recognizing family or community mathematical pracices to support mathematics learning, and finding ways to help students learn and use mathematics to solve authentic problems that affect their lives (Aguirre 2009; Aguirre et al. 2012; Civil 2007; Brenner and Moschkovich 2002; Gutiérrez 2002; Gutstein 2006; Moschkovich 1999; Simic-Mueller, Turner, and Varley 2009; Staats 2009; Turner et al. 2012; Turner and Strawhun 2007).

Mathematics Teaching Practices: Supporting Equitable Mathematics Teaching

Mathematics Teaching Practices	Equitable Teaching
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.	● Establish learning progressions that build students' mathematical understanding, increase their confidence, and support their mathematical identities as doers of mathematics. ● Establish high expectations to ensure that each and every student has the opportunity to meet the mathematical goals. ● Establish classroom norms for participation that position each and every student as a competent mathematics thinker. ● Establish classroom environments that promote learning mathematics as just, equitable, and inclusive.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.	● Engage students in tasks that provide multiple pathways for success and that require reasoning, problem solving, and modeling, thus enhancing each student's mathematical identity and sense of agency. ● Engage students in tasks that are culturally relevant. ● Engage students in tasks that allow them to draw on their funds of knowledge (i.e., the resources that students bring to the classroom, including their home, cultural, and language experiences).
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.	● Connect conceptual understanding with procedural fluency to help students make sense of mathematics and develop a positive disposition toward mathematics. ● Connect conceptual understanding with procedural fluency to reduce mathematical anxiety and position students as mathematical knowers and doers. ● Connect conceptual understanding with procedural fluency to provide students with a wider range of options for entering a task and building mathematical meaning.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build	● Use discourse to elicit students' ideas and strategies and create space for students to interact with peers to value multiple contributions and diminish hierarchical status

shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.	among students (i.e., perceptions of differences in smartness and ability to participate). ● Use discourse to attend to ways in which students position one another as capable or not capable of doing mathematics. ● Make discourse an expected and natural part of mathematical thinking and reasoning, providing students with the space and confidence to ask questions that enhance their own mathematical learning. ● Use discourse as a means to disrupt structures and language that marginalize students.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.	● Pose purposeful questions, then listen to, and understand students' thinking to signal to students that their thinking is valued and makes sense. ● Pose purposeful questions to assign competence to students. Verbally mark students' ideas as interesting or identify an important aspect of students' strategies to position them as competent. ● Be mindful of the fact that the questions that a teacher asks a student and how the teacher follows up on the student's response can support the student's development of a positive mathematical identity and sense of agency as a thinker and doer of mathematics.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematical concepts and procedures and to use them as tools for problem solving.	● Use multiple representations so that students draw on multiple resources of knowledge to position them as competent. ● Use multiple representations to draw on knowledge and experiences related to the resources that students bring to mathematics (culture, contexts, and experiences). ● Use multiple representations to promote the creation and discussion of unique mathematical representations to position students as mathematically competent.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress	● Elicit student thinking and make use of it during a lesson to send positive messages about students' mathematical identities.

toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.	● Make student thinking public, and then choose to elevate a student to a more prominent position in the discussion by identifying his or her idea as worth exploring, to cultivate a positive mathematical identity. ● Promote a classroom culture in which mistakes and errors are viewed as important reasoning opportunities, to encourage a wider range of students to engage in mathematical discussions with their peers and the teacher.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and support to engage in productive struggle as they grapple with mathematical ideas and relationships.	● Allow time for students to engage with mathematical ideas to support perseverance and identity development. ● Hold high expectations, while offering just enough support and scaffolding to facilitate student progress on challenging work, to communicate caring and confidence in students.

Equity-centered Transformative Technology - Lesson Analysis Tool

Equity-centered Transformative Technology	**Question Prompts to Center Equity**
Dimension 1: Access to Inquiry-based Learning- Provide access to dynamic tools to support inquiry, discovery, and deep mathematical sense-making	In what way does the choice of technology tool foster mathematical inquiry and allow equitable access for each and every learner?
Dimension 2: Math Identity through Authorship and Agency- Promote equitable structures and participation to affirm math identity	In what way does the digital tool allow student ownership and authorship to build positive mathematical identities?
Dimension 3: Formative Assessment- Differentiate instruction with real-time feedback using teacher dashboards and/or ease of scanning for scaffolding	In what way does the digital tool promote formative assessment and differentiation to meet learners' needs?
Dimension 4: Empowerment Through Collective Thinking- Collaboration, communication, and connection for social interaction distributes authority by honoring all student ideas	In what way does the digital tool affirm multiple ideas and empower students to collaborate, communicate and build collective knowledge among their peers?
Dimension 5: Amplification of Mathematical and Cognitive Processes- Technology mediated features that amplify the mathematics process with fidelity (Zbiek et al., 2007)	In what way do the features of the digital tool amplify mathematical or cognitive processes?

Core Values Commitment

The College of Education and Human Development is committed to collaboration, ethical leadership, innovation, research-based practice, and social justice. Students are expected to adhere to these principles: http://cehd.gmu.edu/values/.

GMU Policies and Resources for Students

Policies

- Students must adhere to the guidelines of the Mason Honor Code (see https://catalog.gmu.edu/policies/honor-code-system/).

- Students must follow the university policy for Responsible Use of Computing (see https://universitypolicy.gmu.edu/policies/responsible-use-of-computing/).

- Students are responsible for the content of university communications sent to their Mason email account and are required to activate their account and check it regularly. All communication from the university, college, school, and program will be sent to students **solely** through their Mason email account.

- Students with disabilities who seek accommodations in a course must be registered with George Mason University Disability Services. Approved accommodations will begin at the time the written letter from Disability Services is received by the instructor (see https://ds.gmu.edu/).

- Students must silence all sound emitting devices during class unless otherwise authorized by the instructor.

Campus Resources

- Support for submission of assignments to VIA should be directed to viahelp@gmu.edu or https://cehd.gmu.edu/aero/assessments . Questions or concerns regarding use of Blackboard should be directed to https://its.gmu.edu/knowledge-base/blackboard-instructional-technology-support-for-students/.

- For information on student support resources on campus, see https://ctfe.gmu.edu/teaching/student-support-resources-on-campus

Notice of mandatory reporting of sexual assault, sexual harassment, interpersonal violence, and stalking:

As a faculty member, I am designated as a "Non-Confidential Employee," and must report all disclosures of sexual assault, sexual harassment, interpersonal violence, and stalking to Mason's Title IX Coordinator per University Policy 1202. If you wish to speak with someone confidentially, please contact one of Mason's confidential resources, such as Student Support and Advocacy Center (SSAC) at 703-380-1434 or Counseling and Psychological Services (CAPS) at 703-993-2380. You may also seek assistance or support measures from Mason's Title IX Coordinator by calling 703-993-8730, or emailing titleix@gmu.edu.

For additional information on the College of Education and Human Development, please visit our website https://cehd.gmu.edu/students/ .

Course Materials and Student Privacy:

- All course materials posted to Blackboard or other course site are private; by federal law, any materials that identify specific students (via their name, voice, or image) must not be shared with anyone not enrolled in this class.
- Video recordings of class meetings that include audio or visual information from other students are private and must not be shared.
- Live Video Conference Meetings (e.g. Collaborate or Zoom) that include audio or visual information from other students must be viewed privately and not shared with others in your household.
- Some/All of your CEHD synchronous class meetings may be recorded by your instructor to provide necessary information for students in this class. Recordings will be stored on Blackboard [or another secure site] and will only be accessible to students taking this course during this semester.