George Mason University
College of Education and Human Development
Mathematics Education Leadership

MATH 611.6M8 & 6M9 – Geometry and Measurement for K-8 Teachers
3 Credits, Summer 2021
June 1-July 24 Mondays & Wednesdays/4:30-7:10 p.m. Online

Faculty

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Prerequisites/Corequisites

Admission to the Mathematics Education Leadership Master's Degree Program or instructor permission. Enrollment limited to students with a class of Advanced to Candidacy, Graduate, Non-Degree or Senior Plus. Enrollment is limited to Graduate, Non-Degree or Undergraduate level students. Students in a Non-Degree Undergraduate degree may **not** enroll.

University Catalog Course Description

The course explores the foundations of informal measurement and geometry in one, two, and three dimensions. The van Hiele model for geometric learning is used as a framework for how children build their understanding of length, area, volume, angles, and geometric relationships. Visualization, spatial reasoning, and geometric modeling are stressed. As appropriate, transformational geometry, congruence, similarity, and geometric constructions will be discussed.

Course Overview

This course is for future K-8 mathematics teacher specialists will cover the NCTM, Common Core State Standards and Virginia SOL strands in geometry and measurement, especially those in grades 5-8. Special attention will be given to interpreting and assessing students' work and learning.

Course Delivery Method

This course will be delivered online (76% or more) using a synchronous format via Blackboard Learning Management system (LMS) housed in the MyMason portal. You will log in to the Blackboard (Bb) course site using your Mason email name (everything before @masonlive.gmu.edu) and email password. The course site will be available on May 26.

Under no circumstances, may candidates/students participate in online class sessions (either by phone or Internet) while operating motor vehicles. Further, as expected in a face-to-face

class meeting, such online participation requires undivided attention to course content and communication.

Technical Requirements

To participate in this course, students will need to satisfy the following technical requirements:

- High-speed Internet access with standard up-to-date browsers. To get a list of Blackboard's supported browsers see: https://help.blackboard.com/Learn/Student/Getting_Started/Browser_Support#supported-browsers

  To get a list of supported operation systems on different devices see: https://help.blackboard.com/Learn/Student/Getting_Started/Browser_Support#tested-devices-and-operating-systems
- Students must maintain consistent and reliable access to their GMU email and Blackboard, as these are the official methods of communication for this course.
- Students will need a headset microphone for use with the Blackboard Collaborate web conferencing tool. [Delete this sentence if not applicable.]
- Students may be asked to create logins and passwords on supplemental websites and/or to download trial software to their computer or tablet as part of course requirements.
- The following software plug-ins for PCs and Macs, respectively, are available for free download: [Add or delete options, as desire.]
  - Adobe Acrobat Reader: https://get.adobe.com/reader/
  - Windows Media Player: https://support.microsoft.com/en-us/help/14209/get-windows-media-player
  - Apple Quick Time Player: www.apple.com/quicktime/download/

Expectations

- Course Week:
  Our course week will begin on the day that our synchronous meetings take place as indicated on the Schedule of Classes.
- Log-in Frequency:
  Students must actively check the course Blackboard site and their GMU email for communications from the instructor, class discussions, and/or access to course materials at least 4 times per week. In addition, students must log-in for all scheduled online synchronous meetings.
- Participation:
  Students are expected to actively engage in all course activities throughout the semester, which includes viewing all course materials, completing course activities and assignments, and participating in course discussions and group interactions.
- Technical Competence:
  Students are expected to demonstrate competence in the use of all course technology. Students who are struggling with technical components of the course are expected to seek assistance from the instructor and/or College or University technical services.

- Technical Issues:
  Students should anticipate some technical difficulties during the semester and should, therefore, budget their time accordingly. Late work will not be accepted based on individual technical issues.
- Workload:
  Please be aware that this course is **not** self-paced. Students are expected to meet *specific deadlines* and *due dates* listed in the **Class Schedule** section of this syllabus. It is the student's responsibility to keep track of the weekly course schedule of topics, readings, activities and assignments due.
- Instructor Support:
  Students may schedule a one-on-one meeting to discuss course requirements, content or other course-related issues. Those unable to come to a Mason campus can meet with the instructor via telephone or web conference. Students should email the instructor to schedule a one-on-one session, including their preferred meeting method and suggested dates/times.
- Netiquette:
  The course environment is a collaborative space. Experience shows that even an innocent remark typed in the online environment can be misconstrued. Students must always re-read their responses carefully before posting them, so as others do not consider them as personal offenses. *Be positive in your approach with others and diplomatic in selecting your words*. Remember that you are not competing with classmates, but sharing information and learning from others. All faculty are similarly expected to be respectful in all communications.
- Accommodations:
  Online learners who require effective accommodations to ensure accessibility must be registered with George Mason University Disability Services.

Learner Outcomes or Objectives

This course is designed to enable students to do the following:

1. Candidates will develop a comprehensive understanding of an axiomatic system of reasoning, representation and creation of shapes using old and new ways to draw and construct the concepts of measurement and symmetry of structure.
2. Candidates will examine in depth geometry content appropriate for K-8 mathematics teachers, including the use of technology to study geometry and historical connections to geometry.
3. Candidates will explore fundamentals of geometry, congruence and similarity, two-and three- dimensional figures, transformations and isometries, and basic measurement properties of perimeter, area, and volume.
4. Candidates will examine two frameworks, van Hiele's model for geometric learning and geometric Habits of Mind, in order to assess their own progress throughout the course and to discover these models' pedagogical implications on classroom instruction.

Professional Standards (National Council of Teachers of Mathematics)

Upon completion of this course, students will have met the following professional standards:

Standard 1: Content Knowledge (NCTM NCATE Mathematics Content for Elementary Mathematics Specialist *Addendum to the NCTM NCATE Standards 2012)* Effective elementary

mathematics specialists demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, connections, and applications within and among mathematical content domains. Elementary mathematics specialist candidates: 1a) Demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, applications in varied contexts, and connections within and among mathematical domains (Number and Operations, Algebra, Geometry and Measurement, and Statistics and Probability) as outlined in the NCTM CAEP Mathematics Content for Elementary Mathematics Specialist.

To be prepared to support the development of student mathematical proficiency, all elementary mathematics specialists should know the following topics related to geometry and measurement with their content understanding and mathematical practices supported by appropriate technology and concrete models:

C.3.1 Core concepts including angle, parallel, and perpendicular, and principles of Euclidean geometry in two and three dimensions
C.3.2 Transformations including dilations, translations, rotations, reflections, glide reflections; compositions of transformations; and the expression of symmetry and regularity in terms of transformations
C.3.3 Congruence, similarity and scaling, and their development and expression in terms of transformations
C.3.4 Basic geometric figures in one, two, and three dimensions (line segments, lines, rays, circles, arcs, polygons, polyhedral solids, cylinders, cones, and spheres) and their elements (vertices, edges, and faces)
C.3.5 Identification, classification into categories, visualization, two- and three-dimensional representations, and formula rationale and derivation (perimeter, area, and volume) of two- and three-dimensional objects (triangles; classes of quadrilaterals such as rectangles, parallelograms, and trapezoids; regular polygons; rectangular prisms; pyramids; cones; cylinders; and spheres)
C.3.6 Geometric measurement and units (linear, area, surface area, volume, and angle), unit comparison, and the iteration, additivity, and invariance related to measurements
C.3.7 Geometric constructions, axiomatic reasoning, and making and proving conjectures about geometric shapes and relations
C.3.8 Coordinate geometry including the equations of lines and algebraic proofs (e.g., Pythagorean Theorem and its converse)
C.3.9 Historical development and perspectives of geometry and measurement including contributions of significant figures

Standard 2: Mathematical Practices (NCTM NCATE Mathematics Content for Elementary Mathematics Specialist *Addendum to the NCTM NCATE Standards 2012)* Effective elementary mathematics specialists solve problems, represent mathematical ideas, reason, prove, use mathematical models, attend to precision, identify elements of structure, generalize, engage in mathematical communication, and make connections as essential mathematical practices. They understand that these practices intersect with mathematical content and that understanding relies on the ability to demonstrate these practices within and among mathematical domains and in their teaching and mathematics leadership. In their role as teacher, lead teacher, and/or coach/mentor, elementary mathematics specialist candidates:

2a) Use problem solving to develop conceptual understanding, make sense of a wide variety

of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.

2b) Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.

2c) Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts or mathematical problems.

2f) Model how the development of mathematical understanding within and among mathematical domains intersects with the mathematical practices of problem solving, reasoning, communicating, connecting, and representing.

Standard 3: Content Pedagogy (NCTM NCATE Mathematics Content for Elementary Mathematics Specialist *Addendum to the NCTM NCATE Standards 2012)*
Effective elementary mathematics specialists apply knowledge of curriculum standards for mathematics and their relationship to student learning within and across mathematical domains in teaching elementary students and coaching/mentoring elementary classroom teachers. They incorporate research-based mathematical experiences and include multiple instructional strategies and mathematics-specific technological tools in their teaching and coaching/mentoring to develop all students' mathematical understanding and proficiency. As teacher, lead teacher, and coach/mentor, they provide and assist teachers in providing students with opportunities to do mathematics – talking about it and connecting it to both theoretical and real-world contexts. They plan, select, implement, interpret, and assist teachers in the incorporation of formative and summative assessments for monitoring student learning, measuring student mathematical understanding, and informing practice. In their role as teacher, lead teacher, and/or coach/mentor, elementary mathematics specialist candidates:

3a) Apply knowledge of curriculum standards for elementary mathematics and their relationship to student learning within and across mathematical domains in teaching elementary students and coaching/mentoring elementary classroom teachers.

3c) Plan and assist others in planning lessons and units that incorporate a variety of strategies, differentiated instruction for diverse populations, and mathematics-specific and instructional technologies in building all students' conceptual understanding and procedural proficiency.

3e) Implement and promote techniques related to student engagement and communication including selecting high quality tasks, guiding mathematical discussions, identifying key mathematical ideas, identifying and addressing student misconceptions, and employing a range of questioning strategies

Standard 5: Impact on Student Learning (NCTM NCATE Mathematics Content for Elementary Mathematics Specialist *Addendum to the NCTM NCATE Standards 2012)* Elementary mathematics

specialists provide evidence that as a result of their instruction or coaching/mentoring of teachers, elementary students' conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and application of major mathematics concepts in varied contexts have increased. Elementary mathematics specialists support the continual development of a positive disposition toward mathematics. These mathematics specialists show that new student mathematical knowledge has been created as a consequence of their ability to engage students or coach/mentor teachers in mathematical experiences that are developmentally appropriate, require active engagement, and include mathematics-specific technology in building new knowledge. In their role as teacher, lead teacher, and/or coach/mentor, elementary mathematics specialist candidates:

> 5b) Engage students and coach/mentor teachers in using developmentally appropriate mathematical activities and investigations that require active engagement and include mathematics-specific technology in building new knowledge.

Required Texts

Schifter, D., Bastable, V., & Russell, S. J. (2017). *Examining Features of Shape: Casebook.* Reston, VA: NCTM.

Schifter, D., Bastable, V., & Russell, S. J. (2017). *Measuring Space in One, Two, and Three Dimensions: Casebook.* Reston, VA: NCTM.

Recommended Texts

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* NCTM.

Van de Walle, J., Karp, K, & Bay-Williams, J. (2018). *Elementary and middle school mathematics: Teaching developmentally* (10th edition). Pearson Education.

American Psychological Association (2020*). Publication Manual of the American Psychological Association* (7th edition). American Psychological Association.

Course Performance Evaluation

Students are expected to submit all assignments on time in the manner outlined by the instructor (e.g., Blackboard, Tk20, hard copy).

- **Assignments and/or Examinations**

  A. GEOMETRY & MEASUREMENT PORTFOLIO: PART I THE OUT OF SCHOOL VISUAL GEOMETRY PROJECT (15%)
  (*NCTM NCATE 1a, C.3.9*)
  This is a Performance-Based Assessment (PBA). Students will explore an out-of-school connection for either geometry or measurement. Additional details for this assignment (project description & rubric) are provided at the end of the syllabus and in Blackboard/Assignments.

B. GEOMETRY & MEASUREMENT PORTFOLIO: PART II THE CONTENT AND MATH PRACTICES OF GEOMETRY – COLLECTION A (25%)
(*NCTM NCATE 1a, C.3.1-C.3.4, 2a, 2b, 2c, 2f*)
This is a Performance-Based Assessment (PBA). Class sessions will consist of working on rich problems and utilizing the following: practices for promoting productive mathematics discussions, differentiation, the NCTM Process Standards and the multiple representations utilized. Additional details for this assignment (project description & rubric) are provided at the end of the syllabus and in Blackboard/Assignments.

C. GEOMETRY & MEASUREMENT PORTFOLIO: PART III THE CONTENT ANS MATH PRACTICESOF GEOMETRY & MEASUREMENT – COLLECTION B (25%)
(*NCTM NCATE 1a, C.3.5-C.3.8, 2a, 2b, 2c, 2f*)
This is a Performance-Based Assessment (PBA). Class sessions will consist of working on rich problems and utilizing the following: practices for promoting productive mathematics discussions, differentiation, the NCTM Process Standards and the multiple representations utilized. Additional details for this assignment (project description & rubric) are provided at the end of the syllabus and in Blackboard/Assignments.

D. GEOMETRY & MEASUREMENT PORTFOLIO: PART IV EVALUATING GEOMETRY TASKS/LESSONS (25%)
(*NCTM NCATE 3a, 3c, 3e, 5b)*
This is a Performance-Based Assessment (PBA). Students will evaluate a geometry task or lesson. Additional details for this assignment (project description & rubric) are provided at the end of the syllabus and in Blackboard/Assignments.

E. ATTENDANCE & PARTICIPATION (10%)

Attendance: It is your responsibility to attend all class sessions. Please report your reasons for any absences to the instructor in writing.

Tardiness: It is your responsibility to be on time for each class session. Please report your reasons for any tardiness to the instructor in writing.

a) A commitment to participation in class discussions and course depends heavily and primarily on the regular attendance and participation of all involved. Participation will include taking part in discussions informed by critical reading and thinking, leading discussions about selected mathematics problems, and sharing with the class the products of various writing, reflection, lesson planning, and field experience assignments. The expectations, demands, and workload of this course are professional and high.
b) Students are expected to actively participate. This requires students to consider geometry using different strategies and a variety of manipulatives and resources. During math work time, students should be developing algorithms for the entire work time, or discussing and sharing algorithms with each other. During math-talk and discussion times, students should be actively engaged by voicing their thoughts and connecting to topics presented during the discussion.
c) A commitment to reading reflectively and critically the assigned readings. The readings will be used to provide a framework and coherent theme to the course content. They

have been selected to introduce themes in curricular development as well as research and critical commentary on mathematics curriculum.

	LEVEL OF PERFORMANCE			
ELEMENT	***Distinguished*** *(10 points)*	***Proficient*** *(7 - 9 points)*	***Basic*** *(5 - 6 points)*	***Unsatisfactory*** *(0 - 4 points)*
Attendance & Participation	The student attends all classes, is on time, is prepared and follows outlined procedures in case of absence. The student actively participates and continually supports the members of the learning group and the members of the class. Presentations demonstrate a deep knowledge of content as well as implications for teaching.	The student attends most classes, is on time, is prepared and follows outlined procedures in case of absence. The student makes active contributions to the learning group and class. Presentations demonstrate sufficient knowledge of content as well as implications for teaching.	The student is absent for multiple classes and follows outlined procedures in case of absence. At times the student is not prepared for class. Presentations demonstrate minimal knowledge of content and/or implications for teaching.	The student is frequently late for class or absences are not documented by following the outlined procedures. The student is frequently not prepared for class and does not actively participate in discussions. Presentations are lacking knowledge of content and connections to teaching.

- **All assignments require APA formatting:**
  American Psychological Association (2020*). Publication manual of the American psychological association*. American Psychological Association: Washington, DC.

  Specifically, the following aspects of APA formatting should be addressed in any submission:
  - 12 point, Times New Roman font
  - Double spaced
  - Page headers/Running head
  - Cover page with title, author's name and professional affiliation
  - References
  - Headings
  - Citations
  - Clearly organized, grammatically correct, coherent and complete
  - Professional language (i.e. no jargon)

- **Other Requirements**
  Class materials will be posted for each class session on Blackboard. Students are responsible for reviewing these materials and submitting required artifacts (where appropriate) to online class discussion boards.

All assignments are to be turned in to your instructor on time. **Late work will not be accepted for full credit.** Assignments turned in late will receive a 10% deduction from the grade per late day or any fraction thereof (including weekends and holidays).

- **Grading**

15% Portfolio Part I: The Out of School Visual Geometry Project
25% Portfolio Part II: The Content & Math Practices of Geometry (Collection A)
25% Portfolio Part III: The Content & Math Practices of Geometry (Collection B)
25% Portfolio Part IV: Creating A Rich Task
10% Attendance & Participation

GRADING POLICY (Graduate Grading Scale)

A 93%-100%	B+ 87%-89%	C 70%-79%
A- 90%-92%	B 80%-86%	F Below 70%

- **For Master's Degrees**:
  Candidates must have a minimum GPA of 3.00 in coursework presented on the degree application, which may include no more than 6 credits of C. (Grades of C+, C-, or D do not apply to graduate courses. The GPA calculation excludes all transfer courses and Mason non-degree studies credits not formally approved for the degree).

- **For Endorsement Requirements**
  Candidates must have a grade of B or higher for all licensure coursework (endorsement coursework).

Professional Dispositions

Students are expected to exhibit professional behaviors and dispositions at all times. Education professionals are held to high standards, both inside and outside of the classroom. Educators are evaluated on their behaviors and interactions with students, parents, other professionals, and the community at large. At the College of Education and Human Development, dispositions may play a part in the discussions and assignments of any/all courses in a student's program (and thus, as part or all of the grade for those assignments). For additional information visit: https://cehd.gmu.edu/students/polices-procedures/

Class Schedule

EFS = *Examining Features of Shape*
MS 123: *Measuring Space in One, Two and Three Dimensions*

Date	Topic(s)	Readings	Due
Class 1 June 2 **Format** Synchronous	Collaborate Ultra Orientation Syllabus Overview Geometry & Measurement Interactive Notebook GeoGebra Exploration	No Assigned Readings Due	Profile picture and information posted in Collaborate.
Class 2 June 7 **Format** Synchronous	Core Concepts of Geometry and Constructions Learning Phases	EFS: Chapter 1* *Specific case(s) to be identified in class* Chapter 8 (Section 1, Reasoning About Shapes; pp.148-150) MS 123: none *Linking van Hiele Theory to Instruction* Article	van Hiele Level Initial Assessment
Class 3 June 9 **Format** Synchronous	Defining Geometric Figures	EFS: Chapters 2 & 3* *Specific case(s) to be identified in class* Chapter 8 (Section 2, Talking About Shapes; pp. 150-153 Chapter 8 (Section 4, The Complexities of Understanding Angle; pp.155-159) MS 123: None	
Class 4 June 14 **Format** Asynchronous	Portfolio Project Workshop	EFS: None MS 123: None	**PBA DUE TO TK20 Portfolio Part I: The Out of School Visual Geometry Project**
Class 5 June 16 **Format** Synchronous	Identifying, Classifying, and Visualizing in Geometry	EFS: Chapters 4 & 6* *Specific case(s) to be identified in class* Chapter 8 (Section 6, Building and using definitions, pp.162-165) MS 123: None	

Class 6 June 21 **Format** Synchronous	More Geometric Constructions and Making Conjectures	EFS: Chapter 7* *Specific case(s) to be identified in class* MS 123: None	
Class 7 June 23 **Format** Synchronous	Measurement Concepts	EFS: None MS 123: Chapters 1 & 3* *Specific case(s) to be identified in class* Chapter 8 (Section 1, What Is Big? What Is Small? pp.156-158)	
Class 8 June 28 **Format** Asynchronous	Portfolio Project Workshop	EFS: None MS 123: None	**PBA DUE TO TK20 Portfolio Part II: The Content & Math Practices of Geometry (Collection A)**
Class 9 June 30 **Format** Synchronous	Measurement Concepts	EFS: None MS 123: Chapters 4 & 5* *Specific case(s) to be identified in class* Chapter 8 (Section 4, Giving Structure to Space; pp. 164-168)	
Class 10 July 5 **Format** Asynchronous	Transformations and Symmetry	EFS: None MS 123: Chapter 2 (Case 10: Crazy Cakes) *Developing Teachers' Knowledge* Article	
Class 11 July 7 **Format** Synchronous	Portfolio Project Workshop	EFS: None MS 123: None	
Class 12 July 12 **Format** Synchronous	Congruence, Similarity and Scaling	EFS: Chapter 5* *Specific case(s) to be identified in class* Chapter 8 (Section 3, Visualizing Shape; pp.153-155) MS 123: Chapters 6 & 7* *Specific case(s) to be identified in class*	

		Chapter 8 (Section 7, Different Measures, Different Relationships; pp.173-178)	
Class 13 July 14 **Format** Synchronous	Coordinate Geometry and Algebraic Connections	EFS: None MS 123: Chapter 8 (Section 6, Figuring Out Formulas; pp. 171-174) *TRU Framework Conversation Guide*	**PBA DUE TO TK20 Portfolio Part III: The Content & Math Practices of Geometry & Measurement (Collection B)**
Class 14 July 19 **Format** Synchronous	Evaluating Tasks and Lessons	EFS: None *TRU Framework Conversation Guide*	Identify and Bring a Geometry Task/Lesson to Class
Class 15 July 21 **Format** Synchronous	Evaluating Tasks and Lessons	EFS: None MS 123: None	
Class 16 Thursday July 22 **Format** Asynchronous	Portfolio Project Workshop	EFS: None MS 123: None	**PBA DUE TO TK20 Portfolio Part IV: Creating A Rich Task** van Hiele Level Final Assessment

Note: Faculty reserves the right to alter the schedule as necessary, with notification to students.

Core Values Commitment

The College of Education and Human Development is committed to collaboration, ethical leadership, innovation, research-based practice, and social justice. Students are expected to adhere to these principles: http://cehd.gmu.edu/values/.

GMU Policies and Resources for Students

Policies

- Students must adhere to the guidelines of the Mason Honor Code (see https://catalog.gmu.edu/policies/honor-code-system/).

- Students must follow the university policy for Responsible Use of Computing (see https://universitypolicy.gmu.edu/policies/responsible-use-of-computing/).

- Students are responsible for the content of university communications sent to their Mason email account and are required to activate their account and check it regularly. All communication from the university, college, school, and program will be sent to students **solely** through their Mason email account.

- Students with disabilities who seek accommodations in a course must be registered with George Mason University Disability Services. Approved accommodations will begin at the time the written letter from Disability Services is received by the instructor (see https://ds.gmu.edu/).

- Students must silence all sound emitting devices during class unless otherwise authorized by the instructor.

Campus Resources

- Support for submission of assignments to Tk20 should be directed to tk20help@gmu.edu or https://cehd.gmu.edu/aero/tk20. Questions or concerns regarding use of Blackboard should be directed to https://its.gmu.edu/knowledge-base/blackboard-instructional-technology-support-for-students/.

- For information on student support resources on campus, see https://ctfe.gmu.edu/teaching/student-support-resources-on-campus

Notice of mandatory reporting of sexual assault, interpersonal violence, and stalking:
As a faculty member, I am designated as a "Responsible Employee," and must report all disclosures of sexual assault, interpersonal violence, and stalking to Mason's Title IX Coordinator per University Policy 1202. If you wish to speak with someone confidentially, please contact one of Mason's confidential resources, such as Student Support and Advocacy Center (SSAC) at 703-380-1434 or Counseling and Psychological Services (CAPS) at 703-993-2380. You may also seek assistance from Mason's Title IX Coordinator by calling 703-993-8730, or emailing titleix@gmu.edu.

For additional information on the College of Education and Human Development, please visit our website https://cehd.gmu.edu/students/ .

Geometry & Measurement Portfolio Project Description
Course Performance Based Assessment

This is a Performance Based Assessment. The student will collect and annotate a portfolio of geometry principles that reflects core concepts, connections to geometry outside of school, and to the lessons typically taught in K-8 schools. The final product should include the following: 1) an out-of-school mathematics visual project; 2) Part A of the content and practices portfolio; 3) Part B of the content and practices portfolio; and 4) an evaluation of a geometry task/lesson. The final report will be submitted on Blackboard in Tk20.

PART I: OUT-OF-SCHOOL VISUAL GEOMETRY PROJECT

(NCTM Standards C.3.9, 2f, 2d)

Mathematics instruction should draw upon both formal and informal learning experiences. The goal of this project is to identify and describe places or settings where geometry or measurement are useful in hobbies, crafts, leisure activities, home arts, needle arts, sports, games, or other out-of-school activities. "Out-of-school" refers to activities that are traditionally not part of the school curriculum, but those which nevertheless present opportunities for learning mathematics. Often these activities reflect the activities that reflect gender roles or non-Western cultures, which are typically under-represented in formal mathematics. The goal of this project is to bring these important activities forward and connect them to the formal study of geometry. In this project an out-of-school connection for either geometry or measurement is identified. A visual display is included.

Things to consider are:

- **Identification of An Out-of-School Geometry or Measurement Activity:** An out-of-school connection for either geometry or measurement is identified. Information is provided that addresses the following: What is the out-of school geometry or measurement activity? How does the identified activity connect to geometry or measurement content standards (Virginia Mathematics Standards of Learning & Common Core Standards for Mathematics)?
- **Presentation of A Visual Display:** A collection of four images or videos are gathered that depict the out-of-school activity.
- **Annotation of Visual Display:** A description or captions are provided that address the following: How do the visual representations of the activity illustrate the geometric or measurement concept described? Do the written descriptions guide the reader to understand or recognize the geometric or measurement concepts displayed in the images? Does the description reflect precise mathematical vocabulary?
- **Citation of Visual Display:** Each of the images/videos are cited appropriately, even if they belong to the author.
    - For an example of citation please see: https://libguides.tru.ca/c.php?g=714411&p=5093134

PART I: OUT-OF-SCHOOL GEOMETRY OR MEASUREMENT VISUAL				
Level/Criteria	**4**	**3**	**2**	**1**
	Exceeds Expectations	**Meets Expectations**	**Developing**	**Does Not Meet Expectations**
ACTIVITY IDENTIFICATION **NCTM Standard C.3.9** Historical development and perspectives of geometry and measurement including contributions of significant figures and diverse cultures.	The activity identification meets all of the following requirements: -An out-of-school activity is identified that illustrates an informal learning perspective of geometry or measurement. - The out-of-school activity includes a description that clearly and accurately connects to a geometry or measurement concept. -The out-of-school activity includes a description that clearly and accurately connects to the Virginia Mathematics Standards of Learning. -The out-of-school activity includes a description that clearly and accurately connects to the Common Core Standards for Mathematics.	The activity identification meets three of the following requirements: -An out-of-school activity is identified that illustrates an informal learning perspective of geometry or measurement. - The out-of-school activity includes a description that clearly and accurately connects to a geometry or measurement concept. -The out-of-school activity includes a description that clearly and accurately connects to the Virginia Mathematics Standards of Learning. -The out-of-school activity includes a description that clearly and accurately connects to the Common Core Standards for Mathematics.	The activity identification meets two of the following requirements: -An out-of-school activity is identified that illustrates an informal learning perspective of geometry or measurement. - The out-of-school activity includes a description that clearly and accurately connects to a geometry or measurement concept. -The out-of-school activity includes a description that clearly and accurately connects to the Virginia Mathematics Standards of Learning. -The out-of-school activity includes a description that clearly and accurately connects to the Common Core Standards for Mathematics.	The activity identification meets one or fewer of the following requirements: -An out-of-school activity is identified that illustrates an informal learning perspective of geometry or measurement. - The out-of-school activity includes a description that clearly and accurately connects to a geometry or measurement concept. -The out-of-school activity includes a description that clearly and accurately connects to the Virginia Mathematics Standards of Learning. -The out-of-school activity includes a description that clearly and accurately connects to the Common Core Standards for Mathematics.
VISUAL PRESENTATION **NCTM Element 2f** Model how the development of mathematical understanding within and among mathematical domains intersects with the mathematical practices of problem solving, reasoning, communicating, connecting, and representing.	The visual presentation (e.g. Google slides, Prezi, Screencastify, Glog, Popplet) meets all of the following requirements: -Four visual representations (photos or videos) are included. -The four visual representations of the activity clearly illustrate the geometric or measurement concept described. -A static image is displayed even if video is used.	The visual presentation (e.g. Google slides, Prezi, Screencastify, Glog, Popplet) meets two of the following requirements: -Four visual representations (photos or videos) are included. -The four visual representations of the activity clearly illustrate the geometric or measurement concept described. -A static image is displayed even if video is used.	The visual presentation (e.g. Google slides, Prezi, Screencastify, Glog, Popplet) meets one of the following requirements: -Four visual representations (photos or videos) are included. -The four visual representations of the activity clearly illustrate the geometric or measurement concept described. -A static image is displayed even if video is used.	The visual presentation (e.g. Google slides, Prezi, Screencastify, Glog, Popplet) meets none of the following requirements: -Four visual representations (photos or videos) are included. -The four visual representations of the activity clearly illustrate the geometric or measurement concept described. -A static image is displayed even if video is used.

WRITTEN DESCRIPTION **NCTM Element 2d** Organize mathematical thinking and use the language of mathematics to express ideas precisely, both orally and in writing to multiple audiences	The written description meets all of the following requirements: -The written descriptions help the reader understand the images. -The written descriptions help the reader understand the geometric or measurement concepts. -Visuals are clearly labeled. -Descriptions use precise mathematical vocabulary appropriately.	The written description meets three of the following requirements: -The written descriptions help the reader understand the images. -The written descriptions help the reader understand the geometric or measurement concepts. -Visuals are clearly labeled. -Descriptions use precise mathematical vocabulary appropriately.	The written description meets two of the following requirements: -The written descriptions help the reader understand the images. -The written descriptions help the reader understand the geometric or measurement concepts. -Visuals are clearly labeled. -Descriptions use precise mathematical vocabulary appropriately.	The written description meets one or fewer of the following requirements: -The written descriptions help the reader understand the images. -The written descriptions help the reader understand the geometric or measurement concepts. -Visuals are clearly labeled. -Descriptions use precise mathematical vocabulary appropriately.
VISUAL CITATION	The references meet all of the following requirements: - Photo or video #1 is cited. - Photo or video #2 is cited. - Photo or video #3 is cited. - Photo or video #4 is cited. -Citations meet APA formatting guidelines.	The references meet four of the following requirements: - Photo or video #1 is cited. - Photo or video #2 is cited. - Photo or video #3 is cited. - Photo or video #4 is cited. -Citations meet APA formatting guidelines.	The references meet three of the following requirements: - Photo or video #1 is cited. - Photo or video #2 is cited. - Photo or video #3 is cited. - Photo or video #4 is cited. -Citations meet APA formatting guidelines.	The references meet two or fewer of the following requirements: - Photo or video #1 is cited. - Photo or video #2 is cited. - Photo or video #3 is cited. - Photo or video #4 is cited. -Citations meet APA formatting guidelines.

Geometry & Measurement Portfolio Project Description
Course Performance Based Assessment

This is a Performance Based Assessment. The student will collect and annotate a portfolio of geometry principles that reflects core concepts, connections to geometry outside of school, and to the lessons typically taught in K-8 schools. The final product should include the following: 1) an out-of-school mathematics visual project; 2) Part A of the content and practices portfolio; 3) Part B of the content and practices portfolio; and 4) an evaluation of a geometry task. The final report will be submitted on Blackboard in Tk20.

PART II: THE CONTENT & MATH PRACTICES OF GEOMETRY (COLLECTION A)

(NCTM Standards C.3.1, C.3.4, C.3.5, C.3.7, 1a, 2a, 2b, 2c, 2f)

In order to understand the vertical alignment connections, mathematics specialists should have experiences with K-8 geometry standards. In this project students will compile a portfolio of completed investigations (e.g. tasks, constructions) of geometry concepts. Across the collection the student will integrate/demonstrate conceptual and procedural understandings, problem solving, representations, context and the NCTM Process Standards.

Things to consider are:

- **Completed Geometric Investigation (e.g. tasks, constructions):** During each class session students will complete various tasks and/or constructions. Tasks and constructions are included within the portfolio and address the following content standards:
    - *Core Concepts of Geometry and Constructions (Standard C.3.1):* Core concepts including angle, parallel, and perpendicular, and principles of Euclidean geometry in two and three dimensions
    - *Defining Geometric Diagrams (Standard C.3.4):* Basic geometric figures in one, two, and three dimensions (line segments, lines, rays, circles, arcs, polygons, polyhedral solids, cylinders, cones, and spheres) and their elements (vertices, edges, and faces)
    - *Identifying, Classifying, and Visualizing in Geometry (Standard C.3.5):* Identification, classification into categories, visualization, two- and three-dimensional representations, and formula rationale and derivation (perimeter, area, and volume) of two- and three-dimensional objects (triangles; classes of quadrilaterals such as rectangles, parallelograms, and trapezoids; regular polygons; rectangular prisms; pyramids; cones; cylinders; and spheres)
    - *More Geometric Constructions and Making Conjectures (Standard C.3.7):* Geometric constructions, axiomatic reasoning, and making and proving conjectures about geometric shapes and relations
- **Geometric Description - Appearance:** For each task or construction a description of the geometric elements by appearance is provided.
- **Geometric Description - Properties:** For each task or construction a description of relevant mathematical properties is provided.
- **Geometric Relationships & Connections:** For each task or construction, the relationships between and/or within geometric elements are provided.
- **Student Thinking:** For each task or construction, possible student early ideas or notions of the mathematical concept (e.g. student misconceptions) are provided. In this section we are intentional about not using the language of student misconceptions to recognize student thinking as valid for where it falls within the learning trajectory

PART II: THE CONTENT & MATH PRACTICES OF GEOMETRY (COLLECTION A)				
Level/Criteria	**4**	**3**	**2**	**1**
	Exceeds Expectations	**Meets Expectations**	**Developing**	**Does Not Meet Expectations**
GEOMETRY INVESTIGATION: CORE CONCEPTS OF GEOMETRY AND CONSTRUCTIONS **NCTM Standard C.3.1** Core concepts including angle, parallel, and perpendicular, and principles of Euclidean geometry in two and three dimensions	A task or construction that addresses this content standard is included with all of the following elements addressed: -Completed geometric investigation (e.g. task, construction). -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with four of the following elements addressed: -Completed geometric investigation (e.g. task, construction). -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with three of the following elements addressed: -Completed geometric investigation (e.g. task, construction). -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with two or fewer of the following elements addressed: -Completed geometric investigation (e.g. task, construction). -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').

GEOMETRY INVESTIGATION: DEFINING GEOMETRIC FIGURES **NCTM Standard C.3.4** Basic geometric figures in one, two, and three dimensions (line segments, lines, rays, circles, arcs, polygons, polyhedral solids, cylinders, cones, and spheres) and their elements (vertices, edges, and faces)	A task or construction that addresses this content standard is included with all of the following elements addressed: -Completed geometric investigation (e.g. task, construction) -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with four of the following elements addressed: -Completed geometric investigation (e.g. task, construction) -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with three of the following elements addressed: -Completed geometric investigation (e.g. task, construction) -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with two or fewer of the following elements addressed: -Completed geometric investigation (e.g. task, construction) -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').
GEOMETRY INVESTIGATION: IDENTIFYING, CLASSIFYING, AND VISUALIZING GEOMETRY **NCTM Standard C.3.5** Identification, classification into categories, visualization, two- and three-dimensional representations, and formula rationale and derivation (perimeter, area, and volume) of two- and three-dimensional objects (triangles; classes of quadrilaterals such as rectangles, parallelograms, and trapezoids; regular polygons; rectangular prisms; pyramids; cones; cylinders; and spheres)	A task or construction that addresses this content standard is included with all of the following elements addressed: -Completed geometric investigation (e.g. task, construction). -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with four of the following elements addressed: -Completed geometric investigation (e.g. task, construction). -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with three of the following elements addressed: -Completed geometric investigation (e.g. task, construction). -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with two or fewer of the following elements addressed: -Completed geometric investigation (e.g. task, construction). -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').

GEOMETRY INVESTIGATION: MORE CONSTRUCTIONS AND MAKING CONJECTURES **NCTM Standard C.3.7** Geometric constructions, axiomatic reasoning, and making and proving conjectures about geometric shapes and relations	A task or construction that addresses this content standard is included with all of the following elements addressed: -Completed geometric investigation (e.g. task, construction). -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with four of the following elements addressed: -Completed geometric investigation (e.g. task, construction). -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with three of the following elements addressed: -Completed geometric investigation (e.g. task, construction). -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with two or fewer of the following elements addressed: -Completed geometric investigation (e.g. task, construction). -Description of the geometric element(s) by appearance. -Identification of the geometric element(s) by relevant properties. -Identification of relationships either between or within geometric elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').
BUILDING CONCEPTUAL AND PROCEDURAL UNDERSTANDING **NCTM Element 1a** Demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, applications in varied contexts and connections.	Across the collection the candidate demonstrates all of the following elements: -Demonstration of major geometry and measurement concepts. -Development of conceptual knowledge. -Development of procedural knowledge.	Across the collection, the candidate demonstrates two of the following elements: -Demonstration of major geometry and measurement concepts. -Development of conceptual knowledge. -Development of procedural knowledge.	Across the collection, the candidate demonstrates one of the following elements: -Demonstration of major geometry and measurement concepts. -Development of conceptual knowledge. -Development of procedural knowledge.	Across the collection, the candidate demonstrates none of the following elements: -Demonstration of major geometry and measurement concepts. -Development of conceptual knowledge. -Development of procedural knowledge.

PROBLEM SOLVING **NCTM Element 2a** Use problem solving to develop conceptual understanding, make a sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.	Across the collection, the candidate includes all of the following elements: -Application of problem solving to develop an understanding of geometry and measurement concepts. -Sense-making of a variety of problems, tasks and/or constructions. -Application of a variety of methods /strategies. -Creation of generalizations.	Across the collection, the candidate includes three of the following elements: -Application of problem solving to develop an understanding of geometry and measurement concepts -Sense-making of a variety of problems, tasks and/or constructions. -Application of a variety of methods /strategies. -Creation of generalizations.	Across the collection, the candidate includes two of the following elements: -Application of problem solving to develop an understanding of geometry and measurement concepts. -Sense-making of a variety of problems, tasks and/or constructions. -Application of a variety of methods /strategies. -Creation of generalizations.	Across the collection, the candidate includes one or fewer of the following elements: -Application of problem solving to develop an understanding of geometry and measurement concepts. -Sense-making of a variety of problems, tasks and/or constructions. -Application of a variety of methods /strategies. -Creation of generalizations.
REPRESENTATIONS **NCTM Element 2b** Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.	Across the collection, the candidate includes all of the following elements: -Application of multiple representations. -Utilization of appropriate mathematical vocabulary. -Utilization of appropriate mathematical symbols.	Across the collection, the candidate includes two of the following elements: -Application of multiple representations. -Utilization of appropriate mathematical vocabulary. -Utilization of appropriate mathematical symbols.	Across the collection, the candidate includes one of the following elements: -Application of multiple representations. -Utilization of appropriate mathematical vocabulary. -Utilization of appropriate mathematical symbols.	Across the collection, the candidate includes none of the following elements: -Application of multiple representations. -Utilization of appropriate mathematical vocabulary. -Utilization of appropriate mathematical symbols.

CONTEXT **NCTM Element 2c** Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts of mathematical problems.	Across the collection, the candidate includes all of the following elements: -Formulation of mathematical models. -Analysis of mathematical models. -Interpretation of mathematical models.	Across the collection, the candidate includes two of the following elements: -Formulation of mathematical models. -Analysis of mathematical models. -Interpretation of mathematical models.	Across the collection, the candidate includes one of the following elements: -Formulation of mathematical models. -Analysis of mathematical models. -Interpretation of mathematical models.	Across the collection, the candidate includes none of the following elements: -Formulation of mathematical models. -Analysis of mathematical models. -Interpretation of mathematical models.
NCTM PROCESS STANDARDS **NCTM Element 2f** Model how the development of mathematical understanding within and among mathematical domains intersects with the mathematical practices of problem solving, reasoning, communicating, connecting, and representing.	Across the collection, the candidate includes all of the following elements: -Demonstration of problem solving within/across the mathematical domain. -Demonstration of reasoning within/across the mathematical domain. -Demonstration of communication within/across the mathematical domain. -Demonstration of connections within/across the mathematical domain. -Demonstration of representations within/across the mathematical domain.	Across the collection, the candidate includes four of the following elements: -Demonstration of problem solving within/across the mathematical domain. -Demonstration of reasoning within/across the mathematical domain. -Demonstration of communication within/across the mathematical domain. -Demonstration of connections within/across the mathematical domain. -Demonstration of representations within/across the mathematical domain.	Across the collection, the candidate includes three of the following elements: -Demonstration of problem solving within/across the mathematical domain. -Demonstration of reasoning within/across the mathematical domain. -Demonstration of communication within/across the mathematical domain. -Demonstration of connections within/across the mathematical domain. -Demonstration of representations within/across the mathematical domain.	Across the collection, the candidate includes two or fewer of the following elements: -Demonstration of problem solving within/across the mathematical domain. -Demonstration of reasoning within/across the mathematical domain. -Demonstration of communication within/across the mathematical domain. -Demonstration of connections within/across the mathematical domain. -Demonstration of representations within/across the mathematical domain.

Geometry & Measurement Portfolio Project Description
Course Performance Based Assessment

This is a Performance Based Assessment. The student will collect and annotate a portfolio of geometry principles that reflects core concepts, connections to geometry outside of school, and to the lessons typically taught in K-8 schools. The final product should include the following: 1) an out-of-school mathematics visual project; 2) Part A of the content and practices portfolio; 3) Part B of the content and practices portfolio; and 4) an evaluation of a geometry task. The final report will be submitted on Blackboard in Tk20.

PART III: THE CONTENT & MATH PRACTICES OF GEOMETRY & MEASUREMENT (COLLECTION B)

(NCTM Standards C.3.6, C.3.2, C.3.3, C.3.8, 1a, 2a, 2b, 2c, 2f)

In order to understand the vertical alignment connections, mathematics specialists should have experiences with K-8 geometry and measurement standards. In this project students will compile a portfolio of completed investigations (e.g. tasks, constructions) of geometry and measurement concepts. Across the collection the student will integrate/demonstrate conceptual and procedural understandings, problem solving, representations, context and the NCTM Process Standards.

Things to consider are:

- **Completed Geometric & Measurement Investigation (e.g. tasks, constructions):** During each class session students will complete various tasks and/or constructions. Tasks and constructions are included within the portfolio and address the following content standards:
    - *Measurement Concepts (NCTM Standard C.3.6):* Geometric measurement and units (linear, area, surface area, volume, and angle), unit comparison, and the iteration, additivity, and invariance related to measurements
    - *Transformations and Symmetry (NCTM Standard C.3.2):* Transformations including dilations, translations, rotations, reflections, glide reflections; compositions of transformations; and the expression of symmetry and regularity in terms of transformations
    - *Congruence, Similarity, and Scaling (NCTM Standard C.3.3):* Congruence, similarity and scaling, and their development and expression in terms of transformations
    - *Coordinate Geometry and Algebraic Connections (NCTM Standard C.3.8):* Coordinate geometry including the equations of lines and algebraic proofs (e.g., Pythagorean Theorem and its converse)
- **Student Thinking:** For each task or construction, possible early ideas or notions of the mathematical concept (e.g. student misconceptions) are provided.

PART III: THE CONTENT & MATH PRACTICES OF GEOMETRY & MEASUREMENT (COLLECTION B)				
Level/Criteria	**4**	**3**	**2**	**1**
	Exceeds Expectations	**Meets Expectations**	**Developing**	**Does Not Meet Expectations**
GEOMETRY & MEASUREMENT INVESTIGATION: MEASUREMENT CONCEPTS **NCTM Standard C.3.6** Geometric measurement and units (linear, area, surface area, volume, and angle), unit comparison, and the iteration, additivity, and invariance related to measurements	A task or construction that addresses this content standard is included with all of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with three of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with two of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with one or fewer of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').

GEOMETRIC INVESTIGATION: TRANSFORMATIONS AND SYMMETRY **NCTM Standard C.3.2** Transformations including dilations, translations, rotations, reflections, glide reflections; compositions of transformations; and the expression of symmetry and regularity in terms of transformations	A task or construction that addresses this content standard is included with all of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with three of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with two of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with one or fewer of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').
GEOMETRIC INVESTIGATION: CONGRUENCE, SIMILARITY, AND SCALING **NCTM Standard C.3.3** Congruence, similarity and scaling, and their development and expression in terms of transformations	A task or construction that addresses this content standard is included with all of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with three of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with two of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with one or fewer of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').

GEOMETRIC INVESTIGATION: COORDINATE GEOMETRY AND ALGEBRAIC CONNECTIONS **NCTM Standard C.3.8** Coordinate geometry including the equations of lines and algebraic proofs (e.g., Pythagorean Theorem and its converse)	A task or construction that addresses this content standard is included with all of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with three of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with two of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').	A task or construction that addresses this content standard is included with one or fewer of the following elements addressed: -Completed investigation (e.g. task, construction). -Identification of the relevant properties and/or mathematical vocabulary. -Identification of relationships either between or within elements. -Possible early ideas or notions of student thinking (e.g. background knowledge, informal learning, 'misconceptions').
BUILDING CONCEPTUAL AND PROCEDURAL UNDERSTANDING **NCTM Element 1a** Demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, applications in varied contexts and connections within and among mathematical content domains.	Across the collection the candidate demonstrates all of the following elements: -Demonstration of major geometry and measurement concepts. -Development of conceptual knowledge. -Development of procedural knowledge.	Across the collection, the candidate demonstrates two of the following elements: -Demonstration of major geometry and measurement concepts. -Development of conceptual knowledge. -Development of procedural knowledge.	Across the collection, the candidate demonstrates one of the following elements: -Demonstration of major geometry and measurement concepts. -Development of conceptual knowledge. -Development of procedural knowledge.	Across the collection, the candidate demonstrates none of the following elements: -Demonstration of major geometry and measurement concepts. -Development of conceptual knowledge. -Development of procedural knowledge.

PROBLEM SOLVING **NCTM Element 2a** Use problem solving to develop conceptual understanding, make a sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.	Across the collection, the candidate includes all of the following elements: -Application of problem solving to develop an understanding of geometry and measurement concepts. -Sense-making of a variety of problems, tasks and/or constructions. -Application of a variety of methods /strategies. -Creation of generalizations.	Across the collection, the candidate includes three of the following elements: -Application of problem solving to develop an understanding of geometry and measurement concepts. -Sense-making of a variety of problems, tasks and/or constructions. -Application of a variety of methods /strategies. -Creation of generalizations.	Across the collection, the candidate includes two of the following elements: -Application of problem solving to develop an understanding of geometry and measurement concepts. -Sense-making of a variety of problems, tasks and/or constructions. -Application of a variety of methods /strategies. -Creation of generalizations.	Across the collection, the candidate includes one or fewer of the following elements: -Application of problem solving to develop an understanding of geometry and measurement concepts. -Sense-making of a variety of problems, tasks and/or constructions. -Application of a variety of methods /strategies. -Creation of generalizations.
REPRESENTATIONS **NCTM Element 2b** Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.	Across the collection, the candidate includes all of the following elements: -Application of multiple representations. -Utilization of appropriate mathematical vocabulary. -Utilization of appropriate mathematical symbols.	Across the collection, the candidate includes two of the following elements: -Application of multiple representations. -Utilization of appropriate mathematical vocabulary. -Utilization of appropriate mathematical symbols.	Across the collection, the candidate includes one of the following elements: -Application of multiple representations. -Utilization of appropriate mathematical vocabulary. -Utilization of appropriate mathematical symbols.	Across the collection, the candidate includes none of the following elements: -Application of multiple representations. -Utilization of appropriate mathematical vocabulary. -Utilization of appropriate mathematical symbols.

CONTEXT **NCTM Element 2c** Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts of mathematical problems.	Across the collection, the candidate includes all of the following elements: -Formulation of mathematical models, images and/or diagrams. -Analysis of mathematical models. -Interpretation of mathematical models.	Across the collection, the candidate includes two of the following elements: -Formulation of mathematical models, images and/or diagrams. -Analysis of mathematical models. -Interpretation of mathematical models.	Across the collection, the candidate includes one of the following elements: -Formulation of mathematical models, images and/or diagrams. -Analysis of mathematical models. -Interpretation of mathematical models.	Across the collection, the candidate includes none of the following elements: -Formulation of mathematical models, images and/or diagrams. -Analysis of mathematical models. -Interpretation of mathematical models.
NCTM PROCESS STANDARDS **NCTM Element 2f** Model how the development of mathematical understanding within and among mathematical domains intersects with the mathematical practices of problem solving, reasoning, communicating, connecting, and representing.	Across the collection, the candidate includes all of the following elements: -Demonstration of problem solving within/across the mathematical domain. -Demonstration of reasoning within/across the mathematical domain. -Demonstration of communication within/across the mathematical domain. -Demonstration of connections within/across the mathematical domain. -Demonstration of representations within/across the mathematical domain.	Across the collection, the candidate includes four of the following elements: -Demonstration of problem solving within/across the mathematical domain. -Demonstration of reasoning within/across the mathematical domain. -Demonstration of communication within/across the mathematical domain. -Demonstration of connections within/across the mathematical domain. -Demonstration of representations within/across the mathematical domain.	Across the collection, the candidate includes three of the following elements: -Demonstration of problem solving within/across the mathematical domain. -Demonstration of reasoning within/across the mathematical domain. -Demonstration of communication within/across the mathematical domain. -Demonstration of connections within/across the mathematical domain. -Demonstration of representations within/across the mathematical domain.	Across the collection, the candidate includes two or fewer of the following elements: -Demonstration of problem solving within/across the mathematical domain. -Demonstration of reasoning within/across the mathematical domain. -Demonstration of communication within/across the mathematical domain. -Demonstration of connections within/across the mathematical domain. -Demonstration of representations within/across the mathematical domain.

Geometry & Measurement Portfolio Project Description
Course Performance Based Assessment

This is a Performance Based Assessment. The student will collect and annotate a portfolio of geometry principles that reflects core concepts, connections to geometry outside of school, and to the lessons typically taught in K-8 schools. The final product should include the following: 1) an out-of-school mathematics visual project; 2) Part A of the content and practices portfolio; 3) Part B of the content and practices portfolio; and 4) an evaluation of a geometry task. The final report will be submitted on Blackboard in Tk20.

PART IV: EVALUATING GEOMETRY TASKS/LESSONS

(NCTM Standards 3a, 3c, 3e, 5b)

Mathematics instruction should include rich opportunities for student learning. In this project, students will identify and evaluate a geometry task/lesson. Guiding questions for this assignment are adapted from research-informed resources (Driscoll et al., 2016; Schoenfeld, 2018).

Things to consider are:

- **Content Analysis:** The student will identify a task/lesson with a primary learning objective grounded in geometry concepts. The student will align the task/lesson with the corresponding: 1) van Hiele theory of geometric thought level; 2) van Hiele phases of learning level(s) as organized within the lesson; 3) Virginia Mathematics Standards of Learning; 4) Common Core State Standard(s) for Mathematics; and 5) Mathematics Teaching Practice(s) (NCTM, 2014). Guiding questions to consider in this analysis are: In what ways are the van Hiele levels, standards and practices connected? What modifications or considerations will increase the opportunities for students developing a conceptual understanding?
- **Student Thinking Analysis:** The student? will analyze the task/lesson through the lens of promoting student thinking, ownership and agency. Guiding questions to consider in this analysis are: What opportunities do students have to make their own sense of important ideas? How could more opportunities be integrated into this lesson/task?
- **Representation & Communication Analysis:** The student will analyze the task/lesson through the lens of promoting multiple representations and communication. Guiding questions to consider in this analysis are: What representations does the task/lesson promote? What opportunities exist for students to respond to each other's ideas? How could you create more opportunities for students to engage with multiple representations? How could you create more opportunities for students to communicate?
- **Technology Analysis:** The student will analyze the task/lesson through the lens of integrating technology into a synchronous online experience. Guiding questions to consider in this analysis are: What mathematics-specific technology is included to support students building new knowledge? How could you create more opportunities that include mathematics-specific technology?

PORTFOLIO PART IV: EVALUATING GEOMETRY TASKS				
Level/Criteria	**4**	**3**	**2**	**1**
	Exceeds Expectations	**Meets Expectations**	**Developing**	**Does Not Meet Expectations**
CONTENT ANALYSIS **NCTM Element 3a** Apply knowledge of curriculum standards for elementary mathematics and their relationship to student learning within and across mathematical domains in teaching elementary students and coaching/mentoring elementary classroom teachers.	The candidate includes all of the following elements: -A task/lesson with a primary learning objective grounded in geometry or measurement concepts. -Alignment of the task/lesson with van Hiele theory of geometric thought level, phases of learning level(s) as organized within the lesson, Virginia Mathematics Standards of Learning, Common Core State Standard(s) for Mathematics, the Mathematics Teaching Practice(s) (NCTM, 2014). -Connection between the van Hiele levels, standards and practices is clearly described -Identification of modifications or considerations to increase the opportunities for students developing a conceptual understanding	The candidate includes three of the following elements: -A task/lesson with a primary learning objective grounded in geometry or measurement concepts. -Alignment of the task/lesson with van Hiele theory of geometric thought level, phases of learning level(s) as organized within the lesson, Virginia Mathematics Standards of Learning, Common Core State Standard(s) for Mathematics, the Mathematics Teaching Practice(s) (NCTM, 2014). -Connection between the van Hiele levels, standards and practices is clearly described -Identification of modifications or considerations to increase the opportunities for students developing a conceptual understanding	The candidate includes two of the following elements: -A task/lesson with a primary learning objective grounded in geometry or measurement concepts. -Alignment of the task/lesson with van Hiele theory of geometric thought level, phases of learning level(s) as organized within the lesson, Virginia Mathematics Standards of Learning, Common Core State Standard(s) for Mathematics, the Mathematics Teaching Practice(s) (NCTM, 2014). -Connection between the van Hiele levels, standards and practices is clearly described -Identification of modifications or considerations to increase the opportunities for students developing a conceptual understanding	The candidate includes one or fewer of the following elements: -A task/lesson with a primary learning objective grounded in geometry or measurement concepts. -Alignment of the task/lesson with van Hiele theory of geometric thought level, phases of learning level(s) as organized within the lesson, Virginia Mathematics Standards of Learning, Common Core State Standard(s) for Mathematics, the Mathematics Teaching Practice(s) (NCTM, 2014). -Connection between the van Hiele levels, standards and practices is clearly described -Identification of modifications or considerations to increase the opportunities for students developing a conceptual understanding

STUDENT THINKING/EVIDENCE ANALYSIS **NCTM Element 3c** Plan and assist others in planning lessons and units that incorporate a variety of strategies, differentiated instruction for diverse populations, and mathematics-specific and instructional technologies in building all students' conceptual understanding and procedural proficiency	The candidate includes all of the following elements: -Description of the opportunities for students to make their own sense of important ideas - Description of how more opportunities for student sense-making could be integrated into the identified lesson/task -Descriptions of opportunities are clearly articulated. -Descriptions of opportunities are aligned with research-informed teaching practices of geometry	The candidate includes three of the following elements: -Description of the opportunities for students to make their own sense of important ideas - Description of how more opportunities could be integrated into the identified lesson/task -Descriptions of opportunities are clearly articulated. -Descriptions of opportunities are aligned with research-informed teaching practices of geometry	The candidate includes two of the following elements: -Description of the opportunities for students to make their own sense of important ideas - Description of how more opportunities could be integrated into the identified lesson/task -Descriptions of opportunities are clearly articulated. -Descriptions of opportunities are aligned with research-informed teaching practices of geometry	The candidate includes one or fewer of the following elements: -Description of the opportunities for students to make their own sense of important ideas - Description of how more opportunities could be integrated into the identified lesson/task -Descriptions of opportunities are clearly articulated. -Descriptions of opportunities are aligned with research-informed teaching practices of geometry
REPRESENTATION & COMMUNICATION ANALYSIS **NCTM Element 3e** Implement and promote techniques related to student engagement and communication including selecting high quality tasks, guiding mathematical discussions, identifying key mathematical ideas, identifying and addressing student misconceptions, and employing a range of questioning strategies.	The candidate includes all of the following elements: -Identification of representations within the task/lesson. -Identification of opportunities for students to respond to one another's thinking. - Description of how more opportunities could be integrated into the identified lesson/task to engage with multiple representations. - Description of how more opportunities could be integrated into the identified lesson/task for students to communicate. -Descriptions of opportunities are clearly articulated. -Descriptions of opportunities are aligned with research-informed teaching practices of geometry.	The candidate includes five of the following elements: -Identification of representations within the task/lesson. -Identification of opportunities for students to respond to one another's thinking. - Description of how more opportunities could be integrated into the identified lesson/task to engage with multiple representations. - Description of how more opportunities could be integrated into the identified lesson/task for students to communicate -Descriptions of opportunities are clearly articulated. -Descriptions of opportunities are aligned with research-informed teaching practices of geometry.	The candidate includes three to four of the following elements: -Identification of representations within the task/lesson. -Identification of opportunities for students to respond to one another's thinking. - Description of how more opportunities could be integrated into the identified lesson/task to engage with multiple representations. - Description of how more opportunities could be integrated into the identified lesson/task for students to communicate -Descriptions of opportunities are clearly articulated. -Descriptions of opportunities are aligned with research-informed teaching practices of geometry.	The candidate includes two or fewer of the following elements: -Identification of representations within the task/lesson. -Identification of opportunities for students to respond to one another's thinking. - Description of how more opportunities could be integrated into the identified lesson/task to engage with multiple representations. - Description of how more opportunities could be integrated into the identified lesson/task for students to communicate -Descriptions of opportunities are clearly articulated. -Descriptions of opportunities are aligned with research-informed teaching practices of geometry.

TECHNOLOGY ANALYSIS **NCTM Element 5b** Engage students and coach/mentor teachers in using developmentally appropriate mathematical activities and investigations that require active engagement and include mathematics-specific technology in building new knowledge.	The candidate includes all of the following elements: -Description of the opportunities for mathematics-specific technology to support students building new knowledge. - Description of how more opportunities could be included for the integration of mathematics-specific technology to support students building new knowledge. -Descriptions of opportunities are clearly articulated. -Descriptions of opportunities are aligned with research-informed teaching practices of geometry.	The candidate includes three of the following elements: -Description of the opportunities for mathematics-specific technology to support students building new knowledge. - Description of how more opportunities could be included for the integration of mathematics-specific technology to support students building new knowledge. -Descriptions of opportunities are clearly articulated. -Descriptions of opportunities are aligned with research-informed teaching practices of geometry.	The candidate includes two of the following elements: -Description of the opportunities for mathematics-specific technology to support students building new knowledge. - Description of how more opportunities could be included for the integration of mathematics-specific technology to support students building new knowledge. -Descriptions of opportunities are clearly articulated. -Descriptions of opportunities are aligned with research-informed teaching practices of geometry.	The candidate includes one or fewer of the following elements: -Description of the opportunities for mathematics-specific technology to support students building new knowledge. - Description of how more opportunities could be included for the integration of mathematics-specific technology to support students building new knowledge. -Descriptions of opportunities are clearly articulated. -Descriptions of opportunities are aligned with research-informed teaching practices of geometry.
PAPER ORGANIZATION	The paper organization includes all of the following: - A cover page with title, author's name, professional affiliation, and running head. -The paper is well-organized, grammatically correct, coherent, and complete. -The paper has a distinctive focus and voice. -The paper uses professional language (i.e., no jargon). - The paper is double spaced and written in 12-point font, Times New Roman. -The paper meets APA formatting guidelines (headings, citations, references,	The report organization includes five of the following: - A cover page with title, author's name, professional affiliation, and running head. -The paper is well-organized, grammatically correct, coherent, and complete. -The paper has a distinctive focus and voice. -The paper uses professional language (i.e., no jargon). - The paper is double spaced and written in 12-point font, Times New Roman. -The paper meets APA formatting	The report organization includes four of the following: - A cover page with title, author's name, professional affiliation, and running head. -The paper is well-organized, grammatically correct, coherent, and complete. -The paper has a distinctive focus and voice. -The paper uses professional language (i.e., no jargon). - The paper is double spaced and written in 12-point font, Times New Roman. -The paper meets APA formatting	The report organization includes three or fewer of the following: - A cover page with title, author's name, professional affiliation, and running head. -The paper is well-organized, grammatically correct, coherent, and complete. -The paper has a distinctive focus and voice. -The paper uses professional language (i.e., no jargon). - The paper is double spaced and written in 12-point font, Times New Roman. -The paper meets APA formatting guidelines (headings, citations, references,

	running head, page numbers).	guidelines (headings, citations, references, running head, page numbers).	guidelines (headings, citations, references, running head, page numbers).	running head, page numbers).