George Mason University  
College of Education and Human Development  
School of Education  
Mathematics Education Leadership

MATH 610 6M8– Number Systems and Number Theory for K-8 Teachers  
3 Credits, Fall 2020  
Tuesdays, 7:20-10:00 PM Synchronous Online

Faculty
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Email Address: dcrawfo4@gmu.edu

COVID 19 Procedures: Fall 2020
Students, please be aware of and follow all policies and procedures for Mason’s Safe Return to Campus: https://www2.gmu.edu/Safe-Return-Campus

Prerequisites/Corequisites
Admission to the Mathematics Education Leadership Master’s Degree Program or instructor permission.

University Catalog Course Description
This course covers the topics: ways of representing numbers, relationships between numbers, number systems, the meanings of operations and how they relate to one another, and computation within the number system as a foundation for algebra. It also includes episodes in history and development of the number system, and will examine the developmental sequence and learning trajectory as children learn this material.

Course Overview
This course, for future K-8 mathematics teacher specialists, examines concepts contained in the number and operations strands of the Virginia Standards of Learning (SOL), Common Core State Standards (CCSS), and/or referenced in the National Council of Teachers of Mathematics (NCTM) Principles and Standards. Through a coordinated program of activities, participants will learn to
explore the structure of number systems, properties of numbers and develop number sense, computation and estimation concepts and skills.

Course Delivery Method

This course will be delivered online using a synchronous format via Blackboard Learning Management system (LMS) housed in the MyMason portal. You will log in to the Blackboard (Bb) course site using your Mason email name (everything before @masonlive.gmu.edu) and email password. The course site will be available on 8/21/20.

Under no circumstances, may candidates/students participate in online class sessions (either by phone or Internet) while operating motor vehicles. Further, as expected in a face-to-face class meeting, such online participation requires undivided attention to course content and communication.

Technical Requirements

To participate in this course, students will need to satisfy the following technical requirements:

- High-speed Internet access with standard up-to-date browsers. To get a list of Blackboard’s supported browsers see:
  [https://help.blackboard.com/Learn/Student/Getting_Started/Browser_Support#supported-browsers](https://help.blackboard.com/Learn/Student/Getting_Started/Browser_Support#supported-browsers)

To get a list of supported operation systems on different devices see:
  [https://help.blackboard.com/Learn/Student/Getting_Started/Browser_Support#tested-devices-and-operating-systems](https://help.blackboard.com/Learn/Student/Getting_Started/Browser_Support#tested-devices-and-operating-systems)

- Students must maintain consistent and reliable access to their GMU email and Blackboard, as these are the official methods of communication for this course.
- Students will need a headset microphone for use with the Blackboard Collaborate web conferencing tool.
- Students may be asked to create logins and passwords on supplemental websites and/or to download trial software to their computer or tablet as part of course requirements.
- The following software plug-ins for PCs and Macs, respectively, are available for free download:

Expectations
• **Course Week:**
  Our course week will begin on the day that our synchronous meetings take place as indicated on the Schedule of Classes.

• **Log-in Frequency:**
  Students must actively check the course Blackboard site and their GMU email for communications from the instructor, class discussions, and/or access to course materials at least 2 times per week. In addition, students must log-in for all scheduled online synchronous meetings.

• **Participation:**
  Students are expected to actively engage in all course activities throughout the semester, which includes viewing all course materials, completing course activities and assignments, and participating in course discussions and group interactions.

• **Technical Competence:**
  Students are expected to demonstrate competence in the use of all course technology. Students who are struggling with technical components of the course are expected to seek assistance from the instructor and/or College or University technical services.

• **Technical Issues:**
  Students should anticipate some technical difficulties during the semester and should, therefore, budget their time accordingly. Late work will not be accepted based on individual technical issues.

• **Workload:**
  Please be aware that this course is not self-paced. Students are expected to meet specific deadlines and due dates listed in the Class Schedule section of this syllabus. It is the student’s responsibility to keep track of the weekly course schedule of topics, readings, activities and assignments due.

• **Instructor Support:**
  Students may schedule a one-on-one meeting to discuss course requirements, content or other course-related issues. Due to current health recommendations, students do not need to meet in person and can meet with the instructor via telephone or web conference. Students should email the instructor to schedule a one-on-one session, including their preferred meeting method and suggested dates/times.

• **Netiquette:**
  The course environment is a collaborative space. Experience shows that even an innocent remark typed in the online environment can be misconstrued. Students must always re-read their responses carefully before posting them, so as others do not consider them as personal offenses. *Be positive in your approach with others and diplomatic in selecting your words.* Remember that you are not competing with classmates, but sharing information and learning from others. All faculty are similarly expected to be respectful in all communications.

• **Accommodations:**
  Online learners who require effective accommodations to insure accessibility must be registered with George Mason University Disability Services.
Learner Outcomes or Objectives

This course is designed to enable students to do the following:
1. Use numerous representations and conceptual models
2. Develop flexibility in problem solving
3. Explain number concepts and interpret student work in many ways

Professional Standards (National Council of Teachers of Mathematics (NCTM) NCATE Mathematics Content for Elementary Mathematics Specialist (NCATE) Addendum to the NCTM NCATE Standards 2012)

Upon completion of this course, students will have met the following professional standards:

Standard 1: Content Standards
To be prepared to support the development of student mathematical proficiency, all elementary mathematics specialists should know the following topics related to number and operations with their content understanding and mathematical practices supported by appropriate technology and varied representational tools, including concrete models:

C.1.1 Counting and cardinality, comparing and ordering, understanding the structure of the base ten number system with particular attention to place value, order of magnitude, one-to-one correspondence, properties, and relationships in numbers and number systems—whole numbers, integers, rationals, irrationals and reals.
C.1.2 Arithmetic operations (addition, subtraction, multiplication, and division) including mental mathematics and standard and non-standard algorithms, interpretations, and representations of numbers—whole numbers and integers.
C.1.3 Fundamental ideas of number theory—divisors, factors and factorization, multiples, primes and composite numbers.
C.1.5 Historical development and perspectives of number, operations, number systems, and quantity including contributions of significant figures and diverse cultures.

Standard 2: Mathematical Practices (NCTM NCATE Mathematics Content for Elementary Mathematics Specialist Addendum to the NCTM NCATE Standards 2012)
Effective elementary mathematics specialists solve problems, represent mathematical ideas, reason, prove, use mathematical models, attend to precision, identify elements of structure, generalize, engage in mathematical communication, and make connections as essential mathematical practices. They understand that these practices intersect with mathematical content and that understanding relies on the ability to demonstrate these practices within and among mathematical domains and in their teaching and mathematics leadership.

In their role as teacher, lead teacher, and/or coach/mentor, elementary mathematics specialist candidates:
2a) Use problem solving to develop conceptual understanding, make sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.

2b) Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.

2c) Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts or mathematical problems.

2f) Model how the development of mathematical understanding within and among mathematical domains intersects with the mathematical practices of problem solving, reasoning, communicating, connecting, and representing.

Required Texts


Recommended Texts


Course Performance Evaluation

Students are expected to submit all assignments on time in the manner outlined by the instructor (e.g., Blackboard, Tk20, hard copy).

- **Assignments and/or Examinations**
  1. **PARTICIPATION (10%)**

     A commitment to participation in class discussions and course depends heavily and primarily on the regular attendance and participation of all involved. Participation will include taking part in discussions informed by critical reading and thinking, leading discussions about selected mathematics problems, and sharing with the class...
the products of various writing, reflection, lesson planning, and field experience assignments. The expectations, demands and workload of this course are professional and high. This requires students to consider number systems and number theory using different strategies and a variety of manipulatives and resources. During math work time, students should be developing algorithms for the entire work time, or discussing and sharing algorithms with each other. During math-talk and discussion times, students should be actively engaged by voicing their thoughts and connecting to topics presented during the discussion.

A commitment to reading reflectively and critically the assigned readings. The readings will be used to provide a framework and coherent theme to the course content. They have been selected to introduce themes in professional development as well as research and critical commentary on current issues in mathematics education.

Additional information regarding participation, tardies and absences can be found on Blackboard.

1. **NUMBERS & OPERATIONS REFLECTION #1: DEVELOPMENT OF EARLY NUMBER SENSE NUMBER ROUTINE (20%)**

   *(NCTM NCATE 1.1, 2a, 2b, 2c, 2f)*

   This is a Performance-Based Assessment (PBA). The purpose of this Course Performance Based Assessment is for the candidate to demonstrate preparedness to support the development of student mathematical proficiency. All elementary mathematics specialists should know the identified topics related to numbers and operations with their content understanding and mathematical practices. The candidate will implement a number routine centered on development of early number sense in class. Additionally, this assignment requires a written reflection connected to the candidate’s current mathematical understanding and how it has changed. The final product(s) will be submitted on Blackboard in Tk20. For a complete project description, rubric and grading criteria please see assignment descriptions at the end of the syllabus.

1. **NUMBERS & OPERATIONS REFLECTION #2: DEVELOPMENT OF OPERATION NUMBER ROUTINE (20%)**

   *(NCTM NCATE 1.2, 2a, 2b, 2c, 2f)*

   This is a Performance-Based Assessment (PBA). The purpose of this Course Performance Based Assessment is for the candidate to demonstrate preparedness to support the development of student mathematical proficiency. All elementary mathematics specialists should know the identified topics related to numbers and operations with their content understanding and mathematical practices. The candidate will implement a number routine centered on development of operation in class. Additionally, this assignment requires a written reflection connected to the candidate’s current mathematical understanding and how it has changed. The final product(s) will be submitted on Blackboard in Tk20. For a
complete project description, rubric and grading criteria please see assignment descriptions at the end of the syllabus.

1. **NUMBERS & OPERATIONS REFLECTION #3: GROUPWORTHY TASKS IN FUNDAMENTAL IDEAS OF NUMBER THEORY (30%)**
   \((NCTM NCATE 1.3, 2a, 2b, 2c, 2f)\)
   This is a Course Performance Based Assessment. Candidates will collaboratively work in groups to identify K-8 classroom activities that promote the fundamental ideas of number theory: divisors, factors and factorization, multiples, primes and composite numbers. The final product(s) will be submitted on Blackboard in Tk20. For a complete project description, rubric and grading criteria please see assignment descriptions at the end of the syllabus.

1. **NUMBERS & OPERATIONS REFLECTION #4: HISTORICAL IGNITE TALK STYLE VIDEO (20%)**
   \((NCTM NCATE 1.5, 2a, 2b, 2c, 2f)\)
   This is a Course Performance Based Assessment. Candidates will explore the historical development and perspectives of number, operations, number systems, and quantity including contributions of significant figures and diverse cultures. Candidates will identify a non-Eurocentric mathematician who contributed to the key ideas of number systems and number theory, and create a 5-minute Ignite Talk style video on this figure in class. The final product(s) will be submitted on Blackboard in Tk20. For a complete project description, rubric and grading criteria please see assignment descriptions at the end of the syllabus.

- **All assignments require APA formatting:**


  Specifically, the following aspects of APA formatting should be addressed in any submission:
  
  - 12 point, Times New Roman font
  - Double spaced
  - Page headers/Running head
  - Cover page with title, author’s name and professional affiliation
  - References
  - Headings
  - Citations
  - Clearly organized, grammatically correct, coherent and complete
  - Professional language (i.e. no jargon)

- **Other Requirements**
Attendance: It is your responsibility to attend all class sessions. Please report your reasons for any absences to the instructor in writing.

Tardiness: It is your responsibility to be on time for each class session. Please report your reasons for any tardiness to the instructor in writing.

Class materials will be posted for each class session on Blackboard. Students are responsible for reviewing these materials and submitting required artifacts (where appropriate) to online class discussion boards.

All assignments are to be turned in to your instructor on time. **Late work will not be accepted for full credit.** Assignments turned in late will receive a 10% deduction from the grade per late day or any fraction thereof (including weekends and holidays).

- **Grading**
  - A 93%-100%
  - B+ 87%-89%
  - C 70%-79%
  - A- 90%-92%
  - B 80%-86%
  - F Below 70%

  **For Master’s Degrees:**
  Candidates must have a minimum GPA of 3.00 in coursework presented on the degree application, which may include no more than 6 credits of C. (Grades of C+, C-, or D do not apply to graduate courses. The GPA calculation excludes all transfer courses and Mason non-degree studies credits not formally approved for the degree).

  **For Endorsement Requirements**
  Candidates must have a grade of B or higher for all licensure coursework (endorsement coursework).

**Professional Dispositions**
Students are expected to exhibit professional behaviors and dispositions at all times. Education professionals are held to high standards, both inside and outside of the classroom. Educators are evaluated on their behaviors and interactions with students, parents, other professionals, and the community at large. At the College of Education and Human Development, dispositions may play a part in the discussions and assignments of any/all courses in a student’s program (and thus, as part or all of the grade for those assignments). For additional information visit: [https://cehd.gmu.edu/students/polices-procedures/](https://cehd.gmu.edu/students/polices-procedures/)
# Class Schedule

Key: Fosnot & Dolk = Young Mathematicians at work; Number = Developing Essential Understanding of Number; Van de Walle = Elementary and Middle School Mathematics; Cases = Building a system of tens: Casebook; PTA = Principles to action.

<table>
<thead>
<tr>
<th>Date</th>
<th>Topic(s)</th>
<th>Readings</th>
<th>Due</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1 8/25</td>
<td>Syllabus Overview&lt;br&gt; <em>Principles to Actions</em> (NCTM, 2014): The Mathematics Teaching Practices&lt;br&gt; Connecting to the TRU Framework&lt;br&gt; Number Routines</td>
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<tr>
<td>Week 2 9/01</td>
<td>Beliefs of Mathematics Teaching and Learning&lt;br&gt; “Mathematics” or “Mathematizing”?&lt;br&gt; Developing Efficient Computation with Mini Lessons&lt;br&gt; Developing Early Number Concepts and Number Sense</td>
<td>Fosnot &amp; Dolk: Chapter 1 &amp; 7&lt;br&gt; Cases: 6 &amp; 7&lt;br&gt; Supplemental: Number: Big Idea #1&lt;br&gt; Supplemental: Van de Walle: Chapter 8</td>
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<tr>
<td>Week 3 9/08</td>
<td>The role of context in problem solving&lt;br&gt; Mathematical Communities&lt;br&gt; Base-10 structure</td>
<td>Fosnot &amp; Dolk: Chapter 2 &amp; 8&lt;br&gt; Cases: 8 &amp; 9&lt;br&gt; Supplemental: Number: Big Idea #5</td>
<td>Number Sense Number Routine Presentations</td>
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<tr>
<td>Week 4 9/15</td>
<td>Structure of problem solving, shortcuts, and strategy development</td>
<td>Fosnot &amp; Dolk: Cases: 10 &amp; 11</td>
<td>Number Sense Number Routine Presentations</td>
</tr>
</tbody>
</table>
| Week 5  | 9/22 | Written number vs. spoken number | Cases: Chapter 8 - Section 1  
Supplemental: Number: Big Idea #2 & #3 |
|---------|------|---------------------------------|----------------------------------|
| Week 6  | 9/29 | Whole-number Place-value concepts | Cases: 23 -27  
Cases: Chapter 8 – Section 2  
Supplemental: Number: Big Idea #4  
Supplemental: Van de Walle: Chapter 11 |
| Week 7  | 10/6 | Addition and subtraction strategies  
Cassie reviews WIDA | Chapter 4 Excerpt (Horn, 2013)  
Cases: 1 - 3  
Supplemental: Van de Walle: Chapter 12 |
| Week 8  | 10/20| Developing Mathematical Models to Make Meaning of Operations  
Problem solving as a reason for operations  
Addition and subtraction strategies beyond whole numbers | Cases: 13 & 14  
Supplemental: Van de Walle: Chapter 9 |

**Number Sense Number Routine Presentations**

**Number Sense Number Routine Presentations DUE & Uploaded to Tk20**

**Historical Ignite Talk Style Video Due**
<p>| Week 9 10/27 | Fluency with mixed operations - Jigsaw Anxiety and Fear Addition and Subtraction mental strategies (doubles plus one) bay williams article jo boaler origo sangio - order for facts Substitutions for timed test Elevator speech about fact fluency | Fosnot &amp; Dolk: Chapter 5 Supplemental: Number: p43-45 | Operations Number Routine Presentations |
| Week 10 11/03 | 21st century teaching | Fosnot &amp; Dolk: Chapter 3 Cases: 15 &amp; 16 Supplemental: Van de Walle: Chapter 13 Supplemental: Number: p45-46 | Operations Number Routine Presentations |
| Week 11 11/10 | Multiplication Strategies Connecting Division to Multiplication | Fosnot &amp; Dolk: Chapter 4 Cases: 17, 18 Cases: Chapter 8 – Section 4 | Operations Number Routine Presentations |
| Week 12 11/17 | Critical Friend &amp; Group Collaboration | Cases: 19 &amp; 20 Supplemental: Van de Walle: Chapters 1 &amp; 7 | Operations Number Routine Presentations DUE &amp; Uploaded to Tk20 |
| Week 13 11/24 | No Class; University closed in observance of Thanksgiving | | |
| Week 14 | Algorithms Versus Number Sense | Fosnot &amp; Dolk: Chapter 6 | |</p>
<table>
<thead>
<tr>
<th>Date</th>
<th>Topic</th>
<th>Cases</th>
<th>Supplemental</th>
<th>Note</th>
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<tbody>
<tr>
<td>12/01</td>
<td>Problem based classroom</td>
<td>21, 22</td>
<td>Van de Walle: Chapters 3 &amp; 4</td>
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<td></td>
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<td>Supplemental: Number: p49-50</td>
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<td></td>
<td>Groupworthy Tasks</td>
<td>Van de Walle:</td>
<td>Chapter 5</td>
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<tr>
<td>Week 15</td>
<td>Assessments</td>
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<td>Supplemental: Number: Chapter 3</td>
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<tr>
<td>12/08</td>
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<td></td>
<td>Teachers as Mathematicians</td>
<td>Fosnot &amp; Dolk:</td>
<td>Chapter 9</td>
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<tr>
<td>Week 16</td>
<td>Teaching Mathematics</td>
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<td>Supplemental: Van de Walle: Chapter 6</td>
<td><strong>Groupworthy Tasks Project Due &amp; Uploaded to Tk20</strong></td>
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<tr>
<td>12/15</td>
<td>Equitably to All Children</td>
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Note: Faculty reserves the right to alter the schedule as necessary, with notification to students.
Core Values Commitment

The College of Education and Human Development is committed to collaboration, ethical leadership, innovation, research-based practice, and social justice. Students are expected to adhere to these principles: [http://cehd.gmu.edu/values/](http://cehd.gmu.edu/values/).

GMU Policies and Resources for Students

**Policies**

- Students must adhere to the guidelines of the Mason Honor Code (see [https://catalog.gmu.edu/policies/honor-code-system/](https://catalog.gmu.edu/policies/honor-code-system/)).

- Students must follow the university policy for Responsible Use of Computing (see [https://universitypolicy.gmu.edu/policies/responsible-use-of-computing/](https://universitypolicy.gmu.edu/policies/responsible-use-of-computing/)).

- Students are responsible for the content of university communications sent to their Mason email account and are required to activate their account and check it regularly. All communication from the university, college, school, and program will be sent to students solely through their Mason email account.

- Students with disabilities who seek accommodations in a course must be registered with George Mason University Disability Services. Approved accommodations will begin at the time the written letter from Disability Services is received by the instructor (see [https://ds.gmu.edu/](https://ds.gmu.edu/)).

- Students must silence all sound emitting devices during class unless otherwise authorized by the instructor.

**Campus Resources**

- Support for submission of assignments to Tk20 should be directed to tk20help@gmu.edu or [https://cehd.gmu.edu/aero/tk20](https://cehd.gmu.edu/aero/tk20). Questions or concerns regarding use of Blackboard should be directed to [https://its.gmu.edu/knowledge-base/blackboard-instructional-technology-support-for-students/](https://its.gmu.edu/knowledge-base/blackboard-instructional-technology-support-for-students/).

- For information on student support resources on campus, see [https://ctfe.gmu.edu/teaching/student-support-resources-on-campus](https://ctfe.gmu.edu/teaching/student-support-resources-on-campus)

**Notice of mandatory reporting of sexual assault, interpersonal violence, and stalking**
As a faculty member, I am designated as a “Responsible Employee,” and must report all disclosures of sexual assault, interpersonal violence, and stalking to Mason’s Title IX Coordinator per University Policy 1202. If you wish to speak with someone confidentially, please contact one of Mason’s confidential resources, such as Student Support and Advocacy Center (SSAC) at 703-380-1434 or Counseling and Psychological Services (CAPS) at 703-993-2380. You may also seek assistance from Mason’s Title IX Coordinator by calling 703-993-8730, or emailing titleix@gmu.edu.

For additional information on the College of Education and Human Development, please visit our website https://cehd.gmu.edu/students/.
This is a Performance Based Assessment that addresses the following NCTM Math Specialist Standards:

- NCTM Elements: 1a, 2a, 2b, 2c, 2, f
- NCTM Content Standard: C.1.1
  - Counting and cardinality, comparing and ordering, understanding the structure of the base ten number system with particular attention to place value, order of magnitude, one-to-one correspondence, properties, and relationships in numbers and number systems – whole numbers and integers.
- Focus of Reflection: Analyzing and describing development and misconceptions

The purpose of this Course Performance Based Assessment is for the candidate to demonstrate preparedness to support the development of student mathematical proficiency. All elementary mathematics specialists should know the above topics related to numbers and operations with their content understanding and mathematical practices.

The candidate will conduct a math talk and written reflection connected to their current mathematical understanding and how it has changed over the duration of the course. The final product will be submitted on Blackboard in Tk20. For a complete rubric and grading criteria please see the rubric below.

**Summary of Actions**

- Collaborate with your assigned group to develop a math talk. Anticipate a variety (20+) different types of responses and questions to probe diverse responses.
- Conduct a math routine in class as a collaborative team.
- Reflect on your math routine by answering the questions for the written reflection paper.
- Submit the following to Tk20: 1) activity with work samples and/or photos; and 2) written reflection paper.

**WRITTEN REFLECTION PAPER**

The written reflection paper should be clearly written so that the candidate’s understanding of the involved numbers and operations content and mathematical practices is evident. There are five guiding questions that should be addressed in the written reflections:

1. What conceptual and procedural knowledge was required for students to fully understand this math routine? What new knowledge was gained?

2. Describe the development of student sense-making in this math routine. How were your probing questions used along with student responses in this development?
3. How did you and your group anticipate multiple representations? How were mathematical vocabulary, symbols, and generalizations used when connecting different representations?

4. Create or cite an open-ended (multiple ways of solving), multiple solution (multiple correct solutions), contextual (real world) task that is aligned to the mathematical standards of the math routine. In what ways does the math routine support the mathematical understandings of this task?

5. How were the five NCTM Process Standards (Problem Solving, Reasoning & Proof, Representations, Connections & Communications) used in this math routine?

Analyzing and Describing Development and Misconceptions

NCTM Elements: 1a, 2a, 2b, 2c, 2f

<table>
<thead>
<tr>
<th>Number Routines – Development of Early Number Sense</th>
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</thead>
<tbody>
<tr>
<td><strong>NCTM Content Standard</strong>: C.1.1&lt;br&gt;Counting and cardinality, comparing and ordering, understanding the structure of the base ten number system with particular attention to place value, order of magnitude, one-to-one correspondence, properties, and relationships in numbers and number systems – whole numbers and integers.</td>
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<table>
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<tr>
<th><strong>Essential Understanding</strong>: Children develop number sense through a variety of experiences. Mistakes and misconceptions are essential in the development of number sense.</th>
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<tbody>
<tr>
<td>• What is the developmental path for children learning counting and cardinality, comparing and ordering, understanding the structure of the base ten number system with particular attention to place value, order of magnitude, one-to-one correspondence, properties, and relationships in numbers and number systems?</td>
</tr>
<tr>
<td>• What mistakes and misconceptions do children make when developing early number sense?</td>
</tr>
<tr>
<td>• What do these mistakes and misconceptions tell you about a child’s development of early number sense?</td>
</tr>
</tbody>
</table>

Number Routines – Development of Early Number Sense
As a collaborative group, develop a math routine focusing on base 10 and place value. Anticipate a variety (20+) different types of responses and questions to probe diverse responses. Conduct the math talk in class as a collaborative team. Reflect, individually, using the 5 question guide.
## Numbers and Operations Reflection

### Course Performance Based Assessment

<table>
<thead>
<tr>
<th>Level/Criteria</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exceeds Expectations</strong></td>
<td><strong>Meets Expectations</strong></td>
<td><strong>Developing</strong></td>
<td><strong>Does Not Meet Expectations</strong></td>
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</table>

### BUILDING CONCEPTUAL AND PROCEDURAL UNDERSTANDING

**NCTM Element 1.a**

Demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, applications in varied contexts and connections.

<table>
<thead>
<tr>
<th>The candidate includes all of the following elements:</th>
<th>The candidate includes two of the following elements:</th>
<th>The candidate includes one of the following elements:</th>
<th>The candidate does not include any of the following elements:</th>
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<tbody>
<tr>
<td>● Application of conceptual and procedural knowledge in identifying solutions in the problem set</td>
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<td>● Explanation of the development of conceptual to procedural knowledge</td>
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<td>● Discussion of new knowledge gained and the connections to past knowledge and experiences</td>
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### PROBLEM SOLVING

**NCTM Element 2.a**

Use problemsolving to develop conceptual understanding, make a sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.

<table>
<thead>
<tr>
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<td>● Explains how to make sense of the problems in the problem set</td>
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<td>● Make sense of the problems in the problem set</td>
</tr>
<tr>
<td>● Apply a variety of strategies and representations to the problem set</td>
<td>● Apply a variety of strategies and representations to the problem set</td>
<td>● Apply a variety of strategies and representations to the problem set</td>
<td>● Apply a variety of strategies and representations to the problem set</td>
</tr>
</tbody>
</table>

Syllabus Revised August 2020, Template Revised October 2019
<table>
<thead>
<tr>
<th>REPRESENTATIONS NCTM Element 2.b</th>
<th>The candidate includes all of the following elements:</th>
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<th>The candidate includes one of the following elements:</th>
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<td>Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.</td>
<td>Describes how multiple representations were used to model the problem set</td>
<td>Discusses how the representations support the creation of generalizations</td>
<td>Uses appropriate mathematical vocabulary and symbols</td>
<td>Describes how multiple representations were used to model the problem set</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CONTEXT NCTM Element 2.c</th>
<th>The candidate includes all of the following elements:</th>
<th>The candidate includes two of the following elements:</th>
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<tbody>
<tr>
<td>Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts of mathematical problems.</td>
<td>An example of a similar problem with a different context.</td>
<td>An analysis of a similar problem (compare and contrast)</td>
<td>An interpretation of the solution</td>
<td>An example of a similar problem with a different context.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NCTM PROCESS STANDARDS NCTM Element 2.f</th>
<th>The candidate includes a reflection on the process standards that includes a description of how each of the five</th>
<th>The candidate includes a reflection on the process standards that includes a description of how four of the five</th>
<th>The candidate includes a reflection on the process standards that includes a description of how three of the five</th>
<th>The candidate includes a reflection on the process standards that includes a description of how one or two NCTM</th>
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<td>Use and assist teachers in using resources from professional</td>
<td></td>
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| Mathematics education organizations such as teacher/leader discussion groups, teacher networks, and print, digital, and virtual resources/collections | NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using all of the following elements:  
- Teacher/Leader discussion groups  
- Teacher networks  
- Print, digital, and virtual resources/collections | NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using two of the following elements:  
- Teacher/Leader discussion groups  
- Teacher networks  
- Print, digital, and virtual resources/collections | NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using one of the following elements:  
- Teacher/Leader discussion groups  
- Teacher networks  
- Print, digital, and virtual resources/collections | Process Standards impact the mathematical understanding. The reflection does not include any specific instances where the candidate assisted teachers using all of the following elements:  
- Teacher/Leader discussion groups  
- Teacher networks  
- Print, digital, and virtual resources/collections |
Numbers and Operations Reflection 2:
Number Routines – Development of Operation
Course Performance Based Assessment

This is a Performance Based Assessment that addresses the following NCTM Math Specialist Standards:

- NCTM Elements: 1a, 2a, 2b, 2c, 2, f
- NCTM Content Standard: C.1.2
  - Arithmetic operations (addition, subtraction, multiplication, and division) including mental mathematics and standard and non-standard algorithms, interpretations, and representations of numbers—whole numbers and integers.
- Focus of Reflection: Analyzing and describing development and misconceptions of operations

The purpose of this Course Performance Based Assessment is for the candidate to demonstrate preparedness to support the development of student mathematical proficiency. All elementary mathematics specialists should know the above topics related to numbers and operations with their content understanding and mathematical practices.

The candidate will conduct a math talk and written reflection connected to their current mathematical understanding and how it has changed over the duration of the course. The final product will be submitted on Blackboard in Tk20. For a complete rubric and grading criteria please see the rubric below.

Summary of Actions
- Collaborate with your assigned group to develop a math talk. Anticipate a variety (20+) different types of responses and questions to probe diverse responses.
- Conduct a math talk in class as a collaborative team.
- Reflect on your math talk by answering the questions for the written reflection paper.
- Submit the following to Tk20: 1) activity with work samples and/or photos; and 2) written reflection paper.

WRITTEN REFLECTION PAPER
The written reflection paper should be clearly written so that the candidate’s understanding of the involved numbers and operations content and mathematical practices is evident. There are five guiding questions with sub-questions that should be addressed in the written reflections:

1. What conceptual and procedural knowledge was required for students to fully understand this math routine? What new knowledge was gained?

2. Describe the development of student sense-making in this math routine. How were your probing questions used along with student responses in this development?
3. How did you and your group anticipate multiple representations? How were mathematical vocabulary, symbols, and generalizations used when connecting different representations?

4. Create or cite an open-ended (multiple ways of solving), multiple-solution (multiple correct solutions), contextual (real world) task that is aligned to the mathematical standards of the math routine. In what ways does the math routine support the mathematical understandings of this task?

5. How were the five NCTM Process Standards (Problem Solving, Reasoning & Proof, Representations, Connections & Communications) used in this math routine?

Analyzing and Describing Development and Misconceptions

NCTM Elements: 1a, 2a, 2b, 2c, 2f

<table>
<thead>
<tr>
<th>Number Routines – Development of Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NCTM Content Standard:</strong> C.1.2</td>
</tr>
<tr>
<td>Arithmetic operations (addition, subtraction, multiplication, and division) including mental mathematics and standard and non-standard algorithms, interpretations, and representations of numbers – whole numbers and integers.</td>
</tr>
</tbody>
</table>

**Essential Understanding:** Children develop number sense through a variety of experiences. Mistakes and misconceptions are essential in the development of number sense.

- What is the developmental path for children learning arithmetic operations (addition, subtraction, multiplication, and division) including mental mathematics and standard and non-standard algorithms, interpretations, and representations of numbers – whole numbers and integers.
- What mistakes and misconceptions do children make when developing operational sense?
- What do these mistakes and misconceptions tell you about a child’s development of operational sense?

Number Routines – Development of Operational Sense

As a collaborative group, develop a math talk based on addition or subtraction. Anticipate a variety (20+) different types of responses and questions to probe diverse responses. Conduct the math talk in class as a collaborative team. Reflect, individually, using the 5 question guide.
## Numbers and Operations Reflection
### Course Performance Based Assessment

<table>
<thead>
<tr>
<th>Level/Criteria</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exceeds Expectations</strong></td>
<td><strong>Meets Expectations</strong></td>
<td><strong>Developing</strong></td>
<td><strong>Does Not Meet Expectations</strong></td>
<td></td>
</tr>
</tbody>
</table>

### BUILDING CONCEPTUAL AND PROCEDURAL UNDERSTANDING

**NCTM Element 1.a**

Demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, applications in varied contexts and connections.

<table>
<thead>
<tr>
<th>The candidate includes all of the following elements:</th>
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<th>The candidate includes one of the following elements:</th>
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</tr>
</thead>
<tbody>
<tr>
<td>- Application of conceptual and procedural knowledge in identifying solutions in the problem set&lt;br&gt;- Explanation of the development of conceptual to procedural knowledge&lt;br&gt;- Discussion of new knowledge gained and the connections to past knowledge and experiences</td>
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</tr>
</tbody>
</table>

### PROBLEM SOLVING

**NCTM Element 2.a**

Use problem solving to develop conceptual understanding, make a sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.

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<tr>
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<td>- Use of problem solving within the problem set to formulate generalizations&lt;br&gt;- Make sense of the problems in the problem set&lt;br&gt;- Apply a variety of strategies and representations</td>
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</tr>
<tr>
<td>REPRESENTATIONS</td>
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<td>to the problem set</td>
<td>to the problem set</td>
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<tr>
<td>-----------------</td>
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</tbody>
</table>
| **NCTM Element 2.b** | The candidate includes all of the following elements:  
- Describes how multiple representations were used to model the problem set  
- Discusses how the representations support the creation of generalizations  
- Uses appropriate mathematical vocabulary and symbols | The candidate includes two of the following elements:  
- Describes how multiple representations were used to model the problem set  
- Discusses how the representations support the creation of generalizations  
- Uses appropriate mathematical vocabulary and symbols | The candidate includes one of the following elements:  
- Describes how multiple representations were used to model the problem set  
- Discusses how the representations support the creation of generalizations  
- Uses appropriate mathematical vocabulary and symbols | The candidate does not include any of the following elements:  
- Describes how multiple representations were used to model the problem set  
- Discusses how the representations support the creation of generalizations  
- Uses appropriate mathematical vocabulary and symbols |

<table>
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<tr>
<th>CONTEXT</th>
<th>to the problem set</th>
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<th>to the problem set</th>
<th>to the problem set</th>
</tr>
</thead>
</table>
| **NCTM Element 2.c** | The candidate includes all of the following elements:  
- An example of a similar problem with a different context.  
- An analysis of a similar problem (compare and contrast)  
- An interpretation of the solution | The candidate includes two of the following elements:  
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- An analysis of a similar problem (compare and contrast)  
- An interpretation of the solution | The candidate includes one of the following elements:  
- An example of a similar problem with a different context.  
- An analysis of a similar problem (compare and contrast)  
- An interpretation of the solution | The candidate does not include any of the following elements:  
- An example of a similar problem with a different context.  
- An analysis of a similar problem (compare and contrast)  
- An interpretation of the solution |

<table>
<thead>
<tr>
<th>NCTM PROCESS STANDARDS</th>
<th>to the problem set</th>
<th>to the problem set</th>
<th>to the problem set</th>
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</tr>
</thead>
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<tr>
<td><strong>NCTM Element 2.f</strong></td>
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Syllabus Revised August 2020, Template Revised October 2019
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<th>Use and assist teachers in using resources from professional mathematics education organizations such as teacher/leader discussion groups, teacher networks, and print, digital, and virtual resources/collections</th>
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</thead>
<tbody>
<tr>
<td>description of how each of the five NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using all of the following elements:</td>
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<tr>
<td>Teacher/Leader discussion groups</td>
</tr>
<tr>
<td>Teacher networks</td>
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<tr>
<td>Print, digital, and virtual resources/collections</td>
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<tr>
<td>description of how four of the five NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using two of the following elements:</td>
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<td>Teacher/Leader discussion groups</td>
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<tr>
<td>Teacher networks</td>
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<td>Print, digital, and virtual resources/collections</td>
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<tr>
<td>description of how three of the five NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using one of the following elements:</td>
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<tr>
<td>Teacher/Leader discussion groups</td>
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<tr>
<td>description of how one or two NCTM Process Standards impact the mathematical understanding. The reflection does not include any specific instances where the candidate assisted teachers using all of the following elements:</td>
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<tr>
<td>Teacher networks</td>
</tr>
<tr>
<td>Print, digital, and virtual resources/collections</td>
</tr>
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</table>
Numbers Systems and Number Theory Reflection 3:  
Analyzing Math Games  
Course Performance Based Assessment

This is a Performance Based Assessment that addresses the following NCTM Math Specialist Standards:

- NCTM Elements: 1a, 2a, 2b, 2c, 2f
- NCTM Content Standard: C.1.3  
  - Fundamental ideas of number theory – divisors, factors and factorization, multiples, primes and composite numbers.
- Focus of Reflection: Analyzing and aligning math games to math standards and diverse learners

The purpose of this Course Performance Based Assessment is for the candidate to demonstrate preparedness to support the development of student mathematical proficiency. All elementary mathematics specialists should know the above topics related to numbers and operations with their content understanding and mathematical practices.

The candidate will complete an activity and written reflection connected to their current mathematical understanding and how it has changed over the duration of the course. The final product will be submitted on Blackboard in Tk20. For a complete rubric and grading criteria please see the rubric below.

Summary of Actions
- Complete the game activity.
- As a group, reflect on one game by answering the questions for the written reflection paper.
- Each student will submit the following to Tk20: 1) game, work samples, and/or photos; and 2) written reflection paper.

WRITTEN REFLECTION PAPER
The written reflection paper should be clearly written so that the candidate’s understanding of the involved numbers systems and number theory content and mathematical practices is evident. There are five guiding questions with sub-questions that should be addressed in the written reflections:

1. What conceptual or procedural knowledge was gained by engaging in your collection of tasks/games? Describe how the task/game uses strategy to develop the conceptual and procedural knowledge (If it doesn’t, describe how you would modify it so that it does)?
2. How does the use of the task/game promote problem solving, multiple representations, generalizations, connections, and assist the formulation of student generalizations (If it doesn’t, describe how you would modify it so that it does)?
3. How does this task/game support learners to use mathematical modeling, appropriate mathematical vocabulary, and symbols during task/game play (If it doesn’t, describe how you would modify it so that it does)?

4. Create or cite an open-ended (multiple ways of solving), multiple-solution (multiple correct solutions), contextual (real world) task that is aligned to the mathematical standards of the task/game. In what ways does the game support the mathematical understandings of this task?

5. Which of the five NCTM Process Standards (Problem Solving, Reasoning & Proof, Representations, Connections & Communications) was least evident in the task/game. Describe a modification to the game that would enhance this standard.

---

Analyzing and aligning games to mathematical standards and diverse learners

NCTM Elements: 1a, 2a, 2b, 2c, 2f

<table>
<thead>
<tr>
<th>Analyzing Math Games &amp; Groupworthy Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NCTM Content Standard:</strong> C.1.2</td>
</tr>
<tr>
<td>Fundamental ideas of number theory – divisors, factors and factorization, multiples, primes and composite numbers.</td>
</tr>
<tr>
<td><strong>Essential Understanding:</strong> Children develop number theory through a variety of experiences, including games. Games provide an opportunity for students to increase arithmetic fluency while using strategy to analyze patterns and number theory. Mistakes and misconceptions are essential in the development of game strategy and number sense.</td>
</tr>
<tr>
<td>- What mistakes lead to strategies and recognition of number patterns?</td>
</tr>
<tr>
<td>- How aligned is a game to the mathematical standards?</td>
</tr>
<tr>
<td>- How does a math game meet the needs of diverse learners?</td>
</tr>
</tbody>
</table>

**Analyzing Math Games & Groupworthy tasks**

**Phase 1:** Explore and analyze a variety of math games and groupworthy tasks.

As a mathematics specialist, you will need to analyze and evaluate games and activities in order to determine if they meet the instructional goal and the needs of diverse learners. In phase 1, you will explore and analyze a variety of math games. You will use supplemental readings and group collaboration to determine the quality of the games.

**Phase 2:** Identify four math games and collaborate with your assigned group to determine the value of the games.

Each group member will identify four quality math games that meet the following criteria:

*Game 1:* Uses technology
Game 2 - Published by: John Van de Walle, Burns, Fosnot, Illuminations, Math Solutions, Kathy Richardson or other as approved by the instructor

Game 3 - Uses an ELL/SIOP strategy or sentence frame

Game 4 - Uses minimal materials and can be sent home with a child at zero or minimal cost

Phase 3: Game Day
Groups will present their games during a game day. This is not a lecture presentation. All directions must be available for playing without assistance.

Phase 4: Group write-up
Groups will write up the reflection together, and each candidate will submit the reflection to TK-20 individually.
**Numbers Systems and Number Theory Reflection**  
*Course Performance Based Assessment*

<table>
<thead>
<tr>
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**BUILDING CONCEPTUAL AND PROCEDURAL UNDERSTANDING**

**NCTM Element 1.a**  
Demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, applications in varied contexts and connections.

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<td>- Explanation of the development of conceptual to procedural knowledge</td>
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**PROBLEM SOLVING**

**NCTM Element 2.a**  
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<td>- Make sense of the problems in the problem set</td>
</tr>
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<td>- Apply a variety of strategies and representations</td>
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<tr>
<th>NCTM Element 2.b</th>
<th>NCTM Element 2.c</th>
<th>NCTM Element 2.f</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>REPRESENTATIONS</strong></td>
<td><strong>CONTEXT</strong></td>
<td><strong>PROCESS STANDARDS</strong></td>
</tr>
<tr>
<td>Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.</td>
<td>Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts of mathematical problems.</td>
<td>The candidate includes a reflection on the process standards that includes a</td>
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</tbody>
</table>
| The candidate includes all of the following elements:  
  - Describes how multiple representations were used to model the problem set  
  - Discusses how the representations support the creation of generalizations  
  - Uses appropriate mathematical vocabulary and symbols | The candidate includes all of the following elements:  
  - An example of a similar problem with a different context.  
  - An analysis of a similar problem (compare and contrast)  
  - An interpretation of the solution | The candidate includes an analysis on the process standards that includes a |
| The candidate includes two of the following elements:  
  - Describes how multiple representations were used to model the problem set  
  - Discusses how the representations support the creation of generalizations  
  - Uses appropriate mathematical vocabulary and symbols | The candidate includes two of the following elements:  
  - An example of a similar problem with a different context.  
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  - An interpretation of the solution | The candidate includes an analysis on the process standards that includes a |
| The candidate does not include any of the following elements:  
  - Describes how multiple representations were used to model the problem set  
  - Discusses how the representations support the creation of generalizations  
  - Uses appropriate mathematical vocabulary and symbols | The candidate does not include any of the following elements:  
  - An example of a similar problem with a different context.  
  - An analysis of a similar problem (compare and contrast)  
  - An interpretation of the solution | The candidate does not include any of the following elements:  
  - An example of a similar problem with a different context.  
  - An analysis of a similar problem (compare and contrast)  
  - An interpretation of the solution |

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Syllabus Revised August 2020, Template Revised October 2019
| Use and assist teachers in using resources from professional mathematics education organizations such as teacher/leader discussion groups, teacher networks, and print, digital, and virtual resources/collections | description of how each of the five NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using all of the following elements:  
  - Teacher/Leader discussion groups  
  - Teacher networks  
  - Print, digital, and virtual resources/collections | description of how four of the five NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using two of the following elements:  
  - Teacher/Leader discussion groups  
  - Teacher networks  
  - Print, digital, and virtual resources/collections | description of how three of the five NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using one of the following elements:  
  - Teacher/Leader discussion groups  
  - Teacher networks  
  - Print, digital, and virtual resources/collections | description of how one or two NCTM Process Standards impact the mathematical understanding. The reflection does not include any specific instances where the candidate assisted teachers using all of the following elements:  
  - Teacher/Leader discussion groups  
  - Teacher networks  
  - Print, digital, and virtual resources/collections |
Candidates will explore the historical development and perspectives of number, operations, number systems, and quantity including contributions of significant figures and diverse cultures. Candidates will identify a non-Eurocentric mathematician who contributed to the key ideas of number systems and number theory, and present a 5-minute Ignite Talk on this mathematician in class. The final product(s) will be submitted on Blackboard in Tk20.

<table>
<thead>
<tr>
<th>Levels/Criteria</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
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<tbody>
<tr>
<td><strong>NCTM Indicator C.1.5 Historical development of numbers and operations.</strong></td>
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<td>Essay describes the historical development of numbers and operations and provides specific examples.</td>
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<td>Essay includes incomplete description of historical development of numbers and operations.</td>
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<tr>
<td><strong>NCTM Indicator C.1.5 Historical perspectives of numbers and operations.</strong></td>
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