George Mason University
College of Education and Human Development
School of Education
Mathematics Education Leadership

MATH 610 6M8– Number Systems and Number Theory for K-8 Teachers
3 Credits, Fall 2020
Tuesdays, 7:20-10:00 PM Synchronous Online

Facult

Name:	Dr. Deb Crawford
Office Hours:	By Appointment
Office Location:	2400A Thompson Hall, MS 1E8
Office Phone:	540-662-3889 x 88133; 540-664-7495
Email Address:	dcrawfo4@gmu.edu

COVID 19 Procedures: Fall 2020

Students, please be aware of and follow all policies and procedures for Mason's Safe Return to Campus: https://www2.gmu.edu/Safe-Return-Campus

Prerequisites/Corequisites

Admission to the Mathematics Education Leadership Master's Degree Program or instructor permission.

University Catalog Course Description

This course covers the topics: ways of representing numbers, relationships between numbers, number systems, the meanings of operations and how they relate to one another, and computation within the number system as a foundation for algebra. It also includes episodes in history and development of the number system, and will examine the developmental sequence and learning trajectory as children learn this material.

Course Overview

This course, for future K-8 mathematics teacher specialists, examines concepts contained in the number and operations strands of the Virginia Standards of Learning (SOL), Common Core State Standards (CCSS), and/or referenced in the National Council of Teachers of Mathematics (NCTM) Principles and Standards. Through a coordinated program of activities, participants will learn to

explore the structure of number systems, properties of numbers and develop number sense, computation and estimation concepts and skills.

Course Delivery Method

This course will be delivered online using a synchronous format via Blackboard Learning Management system (LMS) housed in the MyMason portal. You will log in to the Blackboard (Bb) course site using your Mason email name (everything before @masonlive.gmu.edu) and email password. The course site will be available on 8/21/20.
Under no circumstances, may candidates/students participate in online class sessions (either by phone or Internet) while operating motor vehicles. Further, as expected in a face-to-face class meeting, such online participation requires undivided attention to course content and communication.

Technical Requirements

To participate in this course, students will need to satisfy the following technical requirements:

- High-speed Internet access with standard up-to-date browsers. To get a list of Blackboard's supported browsers see: https://help.blackboard.com/Learn/Student/Getting_Started/Browser_Support#supported-browsers

  To get a list of supported operation systems on different devices see: https://help.blackboard.com/Learn/Student/Getting_Started/Browser_Support#tested-devices-and-operating-systems
- Students must maintain consistent and reliable access to their GMU email and Blackboard, as these are the official methods of communication for this course.
- Students will need a headset microphone for use with the Blackboard Collaborate web conferencing tool.
- Students may be asked to create logins and passwords on supplemental websites and/or to download trial software to their computer or tablet as part of course requirements.
- The following software plug-ins for PCs and Macs, respectively, are available for free download:
  - Adobe Acrobat Reader: https://get.adobe.com/reader/
  - Windows Media Player: https://support.microsoft.com/en-us/help/14209/get-windows-media-player
  - Apple Quick Time Player: www.apple.com/quicktime/download/

Expectations

- Course Week:
  Our course week will begin on the day that our synchronous meetings take place as indicated on the Schedule of Classes.
- Log-in Frequency:
  Students must actively check the course Blackboard site and their GMU email for communications from the instructor, class discussions, and/or access to course materials at least 2 times per week. In addition, students must log-in for all scheduled online synchronous meetings.
- Participation:
  Students are expected to actively engage in all course activities throughout the semester, which includes viewing all course materials, completing course activities and assignments, and participating in course discussions and group interactions.
- Technical Competence:
  Students are expected to demonstrate competence in the use of all course technology. Students who are struggling with technical components of the course are expected to seek assistance from the instructor and/or College or University technical services.
- Technical Issues:
  Students should anticipate some technical difficulties during the semester and should, therefore, budget their time accordingly. Late work will not be accepted based on individual technical issues.
- Workload:
  Please be aware that this course is **not** self-paced. Students are expected to meet *specific deadlines* and *due dates* listed in the **Class Schedule** section of this syllabus. It is the student's responsibility to keep track of the weekly course schedule of topics, readings, activities and assignments due.
- Instructor Support:
  Students may schedule a one-on-one meeting to discuss course requirements, content or other course-related issues. Due to current health recommendations, students do not need to meet in person and can meet with the instructor via telephone or web conference. Students should email the instructor to schedule a one-on-one session, including their preferred meeting method and suggested dates/times.
- Netiquette:
  The course environment is a collaborative space. Experience shows that even an innocent remark typed in the online environment can be misconstrued. Students must always re-read their responses carefully before posting them, so as others do not consider them as personal offenses. *Be positive in your approach with others and diplomatic in selecting your words*. Remember that you are not competing with classmates, but sharing information and learning from others. All faculty are similarly expected to be respectful in all communications.
- Accommodations:
  Online learners who require effective accommodations to insure accessibility must be registered with George Mason University Disability Services.

Learner Outcomes or Objectives

This course is designed to enable students to do the following:

1. Use numerous representations and conceptual models
2. Develop flexibility in problem solving
3. Explain number concepts and interpret student work in many ways

Professional Standards (National Council of Teachers of Mathematics (NCTM) NCATE Mathematics Content for Elementary Mathematics Specialist (NCATE) *Addendum to the NCTM NCATE Standards 2012*)

Upon completion of this course, students will have met the following professional standards:

Standard 1: Content Standards
To be prepared to support the development of student mathematical proficiency, all elementary mathematics specialists should know the following topics related to number and operations with their content understanding and mathematical practices supported by appropriate technology and varied representational tools, including concrete models:

C.1.1 Counting and cardinality, comparing and ordering, understanding the structure of the base ten number system with particular attention to place value, order of magnitude, one-to-one correspondence, properties, and relationships in numbers and number systems – **whole numbers, integers,** rationals, irrationals and reals.
C.1.2 Arithmetic operations (addition, subtraction, multiplication, and division) including mental mathematics and standard and non-standard algorithms, interpretations, and representations of numbers –whole numbers and integers.
C.1.3 Fundamental ideas of number theory – divisors, factors and factorization, multiples, primes and composite numbers.
C.1.5 Historical development and perspectives of number, operations, number systems, and quantity including contributions of significant figures and diverse cultures.

Standard 2: Mathematical Practices (NCTM NCATE Mathematics Content for Elementary Mathematics Specialist *Addendum to the NCTM NCATE Standards 2012)*
Effective elementary mathematics specialists solve problems, represent mathematical ideas, reason, prove, use mathematical models, attend to precision, identify elements of structure, generalize, engage in mathematical communication, and make connections as essential mathematical practices. They understand that these practices intersect with mathematical content and that understanding relies on the ability to demonstrate these practices within and among mathematical domains and in their teaching and mathematics leadership.

In their role as teacher, lead teacher, and/or coach/mentor, elementary mathematics specialist candidates:

2a) Use problem solving to develop conceptual understanding, make sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.

2b) Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.

2c) Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts or mathematical problems.

2f) Model how the development of mathematical understanding within and among mathematical domains intersects with the mathematical practices of problem solving, reasoning, communicating, connecting, and representing.

Required Texts

Fosnot, C.T., & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division*. Portsmouth, NH: Heinemann.

Schifter, D., Bastable, V., & Russell, S. J. (2016). *Number and operations, part 1: Building a system of tens casebook*. Reston: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: NCTM.

Recommended Texts

American Psychological Association (2010*). Publication Manual of the American Psychological Association* (7th edition). American Psychological Association.

National Council of Teachers of Mathematics (2011). *Developing Essential Understanding of Number and Numeration in Pre-K-Grade 2*. Author.

Van de Walle, J., Karp, K, & Bay-Williams, J.(2018). *Elementary and Middle School Mathematics: Teaching Developmentally*. (10th edition). Pearson Education.

Course Performance Evaluation

Students are expected to submit all assignments on time in the manner outlined by the instructor (e.g., Blackboard, Tk20, hard copy).

- **Assignments and/or Examinations**
  1. **PARTICIPATION (10%)**

     A commitment to participation in class discussions and course depends heavily and primarily on the regular attendance and participation of all involved. Participation will include taking part in discussions informed by critical reading and thinking, leading discussions about selected mathematics problems, and sharing with the class

the products of various writing, reflection, lesson planning, and field experience assignments. The expectations, demands and workload of this course are professional and high. This requires students to consider number systems and number theory using different strategies and a variety of manipulatives and resources. During math work time, students should be developing algorithms for the entire work time, or discussing and sharing algorithms with each other. During math-talk and discussion times, students should be actively engaged by voicing their thoughts and connecting to topics presented during the discussion

A commitment to reading reflectively and critically the assigned readings. The readings will be used to provide a framework and coherent theme to the course content. They have been selected to introduce themes in professional development as well as research and critical commentary on current issues in mathematics education.

Additional information regarding participation, tardies and absences can be found on Blackboard.

1. **NUMBERS & OPERATIONS REFLECTION #1: DEVELOPMENT OF EARLY NUMBER SENSE NUMBER ROUTINE (20%)**
(*NCTM NCATE* 1.1, 2a, 2b, 2c, 2f)
This is a Performance-Based Assessment (PBA). The purpose of this Course Performance Based Assessment is for the candidate to demonstrate preparedness to support the development of student mathematical proficiency. All elementary mathematics specialists should know the identified topics related to numbers and operations with their content understanding and mathematical practices.
The candidate will implement a number routine centered on development of early number sense in class. Additionally, this assignment requires a written reflection connected to the candidate's current mathematical understanding and how it has changed. The final product(s) will be submitted on Blackboard in Tk20. For a complete project description, rubric and grading criteria please see assignment descriptions at the end of the syllabus.

1. **NUMBERS & OPERATIONS REFLECTION #2: DEVELOPMENT OF OPERATION NUMBER ROUTINE (20%)**
(*NCTM NCATE* 1.2, 2a, 2b, 2c, 2f)
This is a Performance-Based Assessment (PBA). The purpose of this Course Performance Based Assessment is for the candidate to demonstrate preparedness to support the development of student mathematical proficiency. All elementary mathematics specialists should know the identified topics related to numbers and operations with their content understanding and mathematical practices.
The candidate will implement a number routine centered on development of operation in class. Additionally, this assignment requires a written reflection connected to the candidate's current mathematical understanding and how it has changed. The final product(s) will be submitted on Blackboard in Tk20. For a

complete project description, rubric and grading criteria please see assignment descriptions at the end of the syllabus.

1. **NUMBERS & OPERATIONS REFLECTION #3: GROUPWORTHY TASKS IN FUNDAMENTAL IDEAS OF NUMBER THEORY (30%)**
(*NCTM NCATE* 1.3, 2a, 2b, 2c, 2f)
This is a Course Performance Based Assessment. Candidates will collaboratively work in groups to identify K-8 classroom activities that promote the fundamental ideas of number theory: divisors, factors and factorization, multiples, primes and composite numbers. The final product(s) will be submitted on Blackboard in Tk20. For a complete project description, rubric and grading criteria please see assignment descriptions at the end of the syllabus.

1. **NUMBERS & OPERATIONS REFLECTION #4: HISTORICAL IGNITE TALK STYLE VIDEO (20%)**
(*NCTM NCATE* 1.5, 2a, 2b, 2c, 2f)
This is a Course Performance Based Assessment. Candidates will explore the historical development and perspectives of number, operations, number systems, and quantity including contributions of significant figures and diverse cultures. Candidates will identify a non-Eurocentric mathematician who contributed to the key ideas of number systems and number theory, and create a 5-minute Ignite Talk style video on this figure in class. The final product(s) will be submitted on Blackboard in Tk20. For a complete project description, rubric and grading criteria please see assignment descriptions at the end of the syllabus.

- **All assignments require APA formatting:**

American Psychological Association (2020*). Publication manual of the American psychological association*. American Psychological Association.

Specifically, the following aspects of APA formatting should be addressed in any submission:

- 12 point, Times New Roman font
- Double spaced
- Page headers/Running head
- Cover page with title, author's name and professional affiliation
- References
- Headings
- Citations
- Clearly organized, grammatically correct, coherent and complete
- Professional language (i.e. no jargon)

- **Other Requirements**

Attendance: It is your responsibility to attend all class sessions. Please report your reasons for any absences to the instructor in writing.

Tardiness: It is your responsibility to be on time for each class session. Please report your reasons for any tardiness to the instructor in writing.

Class materials will be posted for each class session on Blackboard. Students are responsible for reviewing these materials and submitting required artifacts (where appropriate) to online class discussion boards.

All assignments are to be turned in to your instructor on time. **Late work will not be accepted for full credit.** Assignments turned in late will receive a 10% deduction from the grade per late day or any fraction thereof (including weekends and holidays).

- **Grading**

A 93%-100%	B+ 87%-89%	C 70%-79%
A- 90%-92%	B 80%-86%	F Below 70%

For Master's Degrees:
Candidates must have a minimum GPA of 3.00 in coursework presented on the degree application, which may include no more than 6 credits of C. (Grades of C+, C-, or D do not apply to graduate courses. The GPA calculation excludes all transfer courses and Mason non-degree studies credits not formally approved for the degree).

For Endorsement Requirements
Candidates must have a grade of B or higher for all licensure coursework (endorsement coursework).

Professional Dispositions
Students are expected to exhibit professional behaviors and dispositions at all times. Education professionals are held to high standards, both inside and outside of the classroom. Educators are evaluated on their behaviors and interactions with students, parents, other professionals, and the community at large. At the College of Education and Human Development, dispositions may play a part in the discussions and assignments of any/all courses in a student's program (and thus, as part or all of the grade for those assignments). For additional information visit: https://cehd.gmu.edu/students/polices-procedures/

Class Schedule

Key: Fosnot & Dolk = Young Mathematicians at work; Number = Developing Essential Understanding of Number; Van de Walle = Elementary and Middle School Mathematics; Cases = Building a system of tens: Casebook; PTA = Principles to action,

Date	Topic(s)	Readings	Due
Week 1 8/25	Syllabus Overview *Principles to Actions* (NCTM, 2014): The Mathematics Teaching Practices Connecting to the TRU Framework Number Routines		
Week 2 9/01	Beliefs of Mathematics Teaching and Learning "Mathematics" or "Mathematizing"? Developing Efficient Computation with Mini Lessons Developing Early Number Concepts and Number Sense	Fosnot & Dolk: Chapter 1 & 7 Cases: 6 & 7 Supplemental: Number: Big Idea #1 Supplemental: Van de Walle: Chapter 8	
Week 3 9/08	The role of context in problem solving Mathematical Communities Base-10 structure	Fosnot & Dolk: Chapter 2 & 8 Cases: 8 & 9 Supplemental: Number: Big Idea #5	Number Sense Number Routine Presentations
Week 4 9/15	Structure of problem solving, shortcuts, and strategy development	Fosnot & Dolk: Cases: 10 & 11	Number Sense Number Routine Presentations

	Written number vs. spoken number	Cases: Chapter 8 - Section 1 Supplemental: Number: Big Idea #2 & #3	
Week 5 9/22	Whole-number Place-value concepts	Cases: 23 -27 Cases: Chapter 8 – Section 2 Supplemental: Number: Big Idea #4 Supplemental: Van de Walle: Chapter 11	Number Sense Number Routine Presentations
Week 6 9/29	Addition and subtraction strategies Cassie reviews WIDA	Chapter 4 Excerpt (Horn, 2013) Cases: 1 - 3 Supplemental: Van de Walle: Chapter 12	
Week 7 10/06	Addition and subtraction procedures Fact fluency	Cases: 4, 5 & 12 Cases: Chapter 8 – Section 3 Supplemental: Van de Walle: Chapter 10	**Number Sense Number Routine Presentations DUE & Uploaded to Tk20**
Week 8 10/20	Developing Mathematical Models to Make Meaning of Operations Problem solving as a reason for operations Addition and subtraction strategies beyond whole numbers	Cases: 13 & 14 Supplemental: Van de Walle: Chapter 9	**Historical Ignite Talk Style Video Due**

Week 9 10/27	Fluency with mixed operations - Jigsaw Anxiety and Fear Addition and Subtraction mental strategies (doubles plus one) bay williams article jo boaler origo sangio - order for facts Substitutions for timed test Elevator speech about fact fluency	Fosnot & Dolk: Chapter 5 Supplemental: Number: p43-45	Operations Number Routine Presentations
Week 10 11/03	21st century teaching	Fosnot & Dolk: Chapter 3 Cases: 15 & 16 Supplemental: Van de Walle: Chapter 13 Supplemental: Number: p45-46	Operations Number Routine Presentations
Week 11 11/10	Multiplication Strategies Connecting Division to Multiplication	Fosnot & Dolk: Chapter 4 Cases: 17, 18 Cases: Chapter 8 – Section 4	Operations Number Routine Presentations
Week 12 11/17	Critical Friend & Group Collaboration	Cases: 19 & 20 Supplemental: Van de Walle: Chapters 1 & 7	**Operations Number Routine Presentations DUE & Uploaded to Tk20**
Week 13 11/24	No Class; University closed in observance of Thanksgiving		
Week 14	Algorithms Versus Number Sense	Fosnot & Dolk: Chapter 6	

12/01	Problem based classroom	Cases: 21, 22 Supplemental: Van de Walle: Chapters 3 & 4 Supplemental: Number: p49-50	
Week 15 12/08	Groupworthy Tasks Assessments	Van de Walle: Chapter 5 Supplemental: Number: Chapter 3	
Week 16 12/15	Teachers as Mathematicians Teaching Mathematics Equitably to All Children	Fosnot & Dolk: Chapter 9 Supplemental: Van de Walle: Chapter 6	**Groupworthy Tasks Project Due & Uploaded to Tk20**

Note: Faculty reserves the right to alter the schedule as necessary, with notification to students.

Core Values Commitment

The College of Education and Human Development is committed to collaboration, ethical leadership, innovation, research-based practice, and social justice. Students are expected to adhere to these principles: http://cehd.gmu.edu/values/.

GMU Policies and Resources for Students

Policies

- Students must adhere to the guidelines of the Mason Honor Code (see https://catalog.gmu.edu/policies/honor-code-system/).
- Students must follow the university policy for Responsible Use of Computing (see https://universitypolicy.gmu.edu/policies/responsible-use-of-computing/).
- Students are responsible for the content of university communications sent to their Mason email account and are required to activate their account and check it regularly. All communication from the university, college, school, and program will be sent to students **solely** through their Mason email account.
- Students with disabilities who seek accommodations in a course must be registered with George Mason University Disability Services. Approved accommodations will begin at the time the written letter from Disability Services is received by the instructor (see https://ds.gmu.edu/).
- Students must silence all sound emitting devices during class unless otherwise authorized by the instructor.

Campus Resources

- Support for submission of assignments to Tk20 should be directed to tk20help@gmu.edu or https://cehd.gmu.edu/aero/tk20. Questions or concerns regarding use of Blackboard should be directed to https://its.gmu.edu/knowledge-base/blackboard-instructional-technology-support-for-students/.
- For information on student support resources on campus, see https://ctfe.gmu.edu/teaching/student-support-resources-on-campus

Notice of mandatory reporting of sexual assault, interpersonal violence, and stalking:

As a faculty member, I am designated as a "Responsible Employee," and must report all disclosures of sexual assault, interpersonal violence, and stalking to Mason's Title IX Coordinator per University Policy 1202. If you wish to speak with someone confidentially, please contact one of Mason's confidential resources, such as Student Support and Advocacy Center (SSAC) at 703-380-1434 or Counseling and Psychological Services (CAPS) at 703-993-2380. You may also seek assistance from Mason's Title IX Coordinator by calling 703-993-8730, or emailing titleix@gmu.edu.

For additional information on the College of Education and Human Development, please visit our website https://cehd.gmu.edu/students/ .

Numbers and Operations Reflection 1:
Number Routines – Development of Early Number Sense
Course Performance Based Assessment

This is a Performance Based Assessment that addresses the following NCTM Math Specialist Standards:

- NCTM Elements: 1a, 2a, 2b, 2c, 2, f
- NCTM Content Standard: C.1.1
    - Counting and cardinality, comparing and ordering, understanding the structure of the base ten number system with particular attention to place value, order of magnitude, one-to-one correspondence, properties, and relationships in numbers and number systems – whole numbers and integers.
- Focus of Reflection: Analyzing and describing development and misconceptions

The purpose of this Course Performance Based Assessment is for the candidate to demonstrate preparedness to support the development of student mathematical proficiency. All elementary mathematics specialists should know the above topics related to numbers and operations with their content understanding and mathematical practices.

The candidate will conduct a math talk and written reflection connected to their current mathematical understanding and how it has changed over the duration of the course. The final product will be submitted on Blackboard in Tk20. For a complete rubric and grading criteria please see the rubric below.

Summary of Actions

- Collaborate with your assigned group to develop a math talk. Anticipate a variety (20+) different types of responses and questions to probe diverse responses.
- Conduct a math routine in class as a collaborative team.
- Reflect on your math routine by answering the questions for the written reflection paper.
- Submit the following to Tk20: 1) activity with work samples and/or photos; and 2) written reflection paper.

WRITTEN REFLECTION PAPER
The written reflection paper should be clearly written so that the candidate's understanding of the involved numbers and operations content and mathematical practices is evident. There are five guiding questions that should be addressed in the written reflections:

1. What conceptual and procedural knowledge was required for students to fully understand this math routine? What new knowledge was gained?

2. Describe the development of student sense-making in this math routine. How were your probing questions used along with student responses in this development?

3. How did you and your group anticipate multiple representations? How were mathematical vocabulary, symbols, and generalizations used when connecting different representations?

4. Create or cite an open-ended (multiple ways of solving), multiple-solution (multiple correct solutions), contextual (real world) task that is aligned to the mathematical standards of the math routine. In what ways does the math routine support the mathematical understandings of this task?

5. How were the five NCTM Process Standards (Problem Solving, Reasoning & Proof, Representations, Connections & Communications) used in this math routine?

Analyzing and Describing Development and Misconceptions

NCTM Elements: 1a, 2a, 2b, 2c, 2f

Number Routines – Development of Early Number Sense
NCTM Content Standard: C.1.1 Counting and cardinality, comparing and ordering, understanding the structure of the base ten number system with particular attention to place value, order of magnitude, one-to-one correspondence, properties, and relationships in numbers and number systems – whole numbers and integers.
Essential Understanding: Children develop number sense through a variety of experiences. Mistakes and misconceptions are essential in the development of number sense. • What is the developmental path for children learning counting and cardinality, comparing and ordering, understanding the structure of the base ten number system with particular attention to place value, order of magnitude, one-to-one correspondence, properties, and relationships in numbers and number systems? • What mistakes and misconceptions do children make when developing early number sense? • What do these mistakes and misconceptions tell you about a child's development of early number sense?

Number Routines – Development of Early Number Sense

As a collaborative group, develop a math routine focusing on base 10 and place value. Anticipate a variety (20+) different types of responses and questions to probe diverse responses. Conduct the math talk in class as a collaborative team. Reflect, individually, using the 5 question guide.

Numbers and Operations Reflection
Course Performance Based Assessment

Level/Criteria	4	3	2	1
	Exceeds Expectations	**Meets Expectations**	**Developing**	**Does Not Meet Expectations**
BUILDING CONCEPTUAL AND PROCEDURAL UNDERSTANDING **NCTM Element 1.a** Demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, applications in varied contexts and connections.	The candidate includes all of the following elements: ● Application of conceptual and procedural knowledge in identifying solutions in the problem set ● Explanation of the development of conceptual to procedural knowledge ● Discussion of new knowledge gained and the connections to past knowledge and experiences	The candidate includes two of the following elements: ● Application of conceptual and procedural knowledge in identifying solutions in the problem set ● Explanation of the development of conceptual to procedural knowledge ● Discussion of new knowledge gained and the connections to past knowledge and experiences	The candidate includes one of the following elements: ● Application of conceptual and procedural knowledge in identifying solutions in the problem set ● Explanation of the development of conceptual to procedural knowledge ● Discussion of new knowledge gained and the connections to past knowledge and experiences	The candidate does not include any of following elements: ● Application of conceptual and procedural knowledge in identifying solutions in the problem set ● Explanation of the development of conceptual to procedural knowledge ● Discussion of new knowledge gained and the connections to past knowledge and experiences
PROBLEM SOLVING **NCTM Element 2.a** Use problem solving to develop conceptual understanding, make a sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.	The candidate includes all of the following elements: ● Describes the use of problem solving within the problem set to formulate generalizations ● Explains how to make sense of the problems in the problem set ● Apply a variety of strategies and representations to the problem set	The candidate includes two of the following elements: ● Use of problem solving within the problem set to formulate generalizations ● Make sense of the problems in the problem set ● Apply a variety of strategies and representations to the problem set	The candidate includes one of the following elements: ● Use of problem solving within the problem set to formulate generalizations ● Make sense of the problems in the problem set ● Apply a variety of strategies and representations to the problem set	The candidate does not include any of following elements: ● Use of problem solving within the problem set to formulate generalizations ● Make sense of the problems in the problem set ● Apply a variety of strategies and representations to the problem set

REPRESENTATIONS **NCTM Element 2.b** Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.	The candidate includes all of the following elements: ● Describes how multiple representations were used to model the problem set ● Discusses how the representations support the creation of generalizations ● Uses appropriate mathematical vocabulary and symbols	The candidate includes two of the following elements: ● Describes how multiple representations were used to model the problem set ● Discusses how the representations support the creation of generalizations ● Uses appropriate mathematical vocabulary and symbols	The candidate includes one of the following elements: ● Describes how multiple representations were used to model the problem set ● Discusses how the representations support the creation of generalizations ● Uses appropriate mathematical vocabulary and symbols	The candidate does not include any of following elements: ● Describes how multiple representations were used to model the problem set ● Discusses how the representations support the creation of generalizations ● Uses appropriate mathematical vocabulary and symbols
CONTEXT **NCTM Element 2.c** Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts of mathematical problems.	The candidate includes all of the following elements: ● An example of a similar problem with a different context. ● An analysis of a similar problem (compare and contrast) ● An interpretation of the solution	The candidate includes two of the following elements: ● An example of a similar problem with a different context. ● An analysis of a similar problem (compare and contrast) ● An interpretation of the solution	The candidate includes one of the following elements: ● An example of a similar problem with a different context. ● An analysis of a similar problem (compare and contrast) ● An interpretation of the solution	The candidate does not include any of following elements: ● An example of a similar problem with a different context. ● An analysis of a similar problem (compare and contrast) ● An interpretation of the solution
NCTM PROCESS STANDARDS **NCTM Element 2.f** Use and assist teachers in using resources from professional	The candidate includes a reflection on the process standards that includes a description of how each of the five	The candidate includes a reflection on the process standards that includes a description of how four of the five	The candidate includes a reflection on the process standards that includes a description of how three of the five	The candidate includes a reflection on the process standards that includes a description of how one or two NCTM

mathematics education organizations such as teacher/leader discussion groups, teacher networks, and print, digital, and virtual resources/ collections	NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using all of the following elements: ● Teacher/Leader discussion groups ● Teacher networks ● Print, digital, and virtual resources/ collections	NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using two of the following elements: ● Teacher/Leader discussion groups ● Teacher networks ● Print, digital, and virtual resources/ collections	NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using one of the following elements: ● Teacher/Leader discussion groups ● Teacher networks ● Print, digital, and virtual resources/ collections	Process Standards impact the mathematical understanding. The reflection does not include any specific instances where the candidate assisted teachers using all of the following elements: ● Teacher/Leader discussion groups ● Teacher networks ● Print, digital, and virtual resources/ collections

Numbers and Operations Reflection 2:
Number Routines – Development of Operation
Course Performance Based Assessment

This is a Performance Based Assessment that addresses the following NCTM Math Specialist Standards:

- NCTM Elements: 1a, 2a, 2b, 2c, 2, f
- NCTM Content Standard: C.1.2
    - Arithmetic operations (addition, subtraction, multiplication, and division) including mental mathematics and standard and non-standard algorithms, interpretations, and representations of numbers –whole numbers and integers.
- Focus of Reflection: Analyzing and describing development and misconceptions of operations

The purpose of this Course Performance Based Assessment is for the candidate to demonstrate preparedness to support the development of student mathematical proficiency. All elementary mathematics specialists should know the above topics related to numbers and operations with their content understanding and mathematical practices.

The candidate will conduct a math talk and written reflection connected to their current mathematical understanding and how it has changed over the duration of the course. The final product will be submitted on Blackboard in Tk20. For a complete rubric and grading criteria please see the rubric below.

Summary of Actions

- Collaborate with your assigned group to develop a math talk. Anticipate a variety (20+) different types of responses and questions to probe diverse responses.
- Conduct a math talk in class as a collaborative team.
- Reflect on your math talk by answering the questions for the written reflection paper.
- Submit the following to Tk20: 1) activity with work samples and/or photos; and 2) written reflection paper.

WRITTEN REFLECTION PAPER

The written reflection paper should be clearly written so that the candidate's understanding of the involved numbers and operations content and mathematical practices is evident. There are five guiding questions with sub-questions that should be addressed in the written reflections:

1. What conceptual and procedural knowledge was required for students to fully understand this math routine? What new knowledge was gained?

2. Describe the development of student sense-making in this math routine. How were your probing questions used along with student responses in this development?

3. How did you and your group anticipate multiple representations? How were mathematical vocabulary, symbols, and generalizations used when connecting different representations?

4. Create or cite an open-ended (multiple ways of solving), multiple-solution (multiple correct solutions), contextual (real world) task that is aligned to the mathematical standards of the math routine. In what ways does the math routine support the mathematical understandings of this task?

5. How were the five NCTM Process Standards (Problem Solving, Reasoning & Proof, Representations, Connections & Communications) used in this math routine?

Analyzing and Describing Development and Misconceptions

NCTM Elements: 1a, 2a, 2b, 2c, 2f

Number Routines – Development of Operation
NCTM Content Standard: C.1.2 Arithmetic operations (addition, subtraction, multiplication, and division) including mental mathematics and standard and non-standard algorithms, interpretations, and representations of numbers –whole numbers and integers.
Essential Understanding: Children develop number sense through a variety of experiences. Mistakes and misconceptions are essential in the development of number sense. • What is the developmental path for children learning arithmetic operations (addition, subtraction, multiplication, and division) including mental mathematics and standard and non-standard algorithms, interpretations, and representations of numbers – whole numbers and integers. • What mistakes and misconceptions do children make when developing operational sense? • What do these mistakes and misconceptions tell you about a child's development of operational sense?

Number Routines – Development of Operational Sense

As a collaborative group, develop a math talk based on addition or subtraction. Anticipate a variety (20+) different types of responses and questions to probe diverse responses. Conduct the math talk in class as a collaborative team. Reflect, individually, using the 5 question guide.

Numbers and Operations Reflection

Course Performance Based Assessment

Level/Criteria	4	3	2	1
	Exceeds Expectations	**Meets Expectations**	**Developing**	**Does Not Meet Expectations**
BUILDING CONCEPTUAL AND PROCEDURAL UNDERSTANDING **NCTM Element 1.a** Demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, applications in varied contexts and connections.	The candidate includes all of the following elements: ● Application of conceptual and procedural knowledge in identifying solutions in the problem set ● Explanation of the development of conceptual to procedural knowledge ● Discussion of new knowledge gained and the connections to past knowledge and experiences	The candidate includes two of the following elements: ● Application of conceptual and procedural knowledge in identifying solutions in the problem set ● Explanation of the development of conceptual to procedural knowledge ● Discussion of new knowledge gained and the connections to past knowledge and experiences	The candidate includes one of the following elements: ● Application of conceptual and procedural knowledge in identifying solutions in the problem set ● Explanation of the development of conceptual to procedural knowledge ● Discussion of new knowledge gained and the connections to past knowledge and experiences	The candidate does not include any of following elements: ● Application of conceptual and procedural knowledge in identifying solutions in the problem set ● Explanation of the development of conceptual to procedural knowledge ● Discussion of new knowledge gained and the connections to past knowledge and experiences
PROBLEM SOLVING **NCTM Element 2.a** Use problem solving to develop conceptual understanding, make a sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.	The candidate includes all of the following elements: ● Describes the use of problem solving within the problem set to formulate generalizations ● Explains how to make sense of the problems in the problem set ● Apply a variety of strategies and representations	The candidate includes two of the following elements: ● Use of problem solving within the problem set to formulate generalizations ● Make sense of the problems in the problem set ● Apply a variety of strategies and representations	The candidate includes one of the following elements: ● Use of problem solving within the problem set to formulate generalizations ● Make sense of the problems in the problem set ● Apply a variety of strategies and representations	The candidate does not include any of following elements: ● Use of problem solving within the problem set to formulate generalizations ● Make sense of the problems in the problem set ● Apply a variety of strategies and representations

	to the problem set	to the problem set	to the problem set	to the problem set
REPRESENTATIONS **NCTM Element 2.b** Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.	The candidate includes all of the following elements: ● Describes how multiple representations were used to model the problem set ● Discusses how the representations support the creation of generalizations ● Uses appropriate mathematical vocabulary and symbols	The candidate includes two of the following elements: ● Describes how multiple representations were used to model the problem set ● Discusses how the representations support the creation of generalizations ● Uses appropriate mathematical vocabulary and symbols	The candidate includes one of the following elements: ● Describes how multiple representations were used to model the problem set ● Discusses how the representations support the creation of generalizations ● Uses appropriate mathematical vocabulary and symbols	The candidate does not include any of following elements: ● Describes how multiple representations were used to model the problem set ● Discusses how the representations support the creation of generalizations ● Uses appropriate mathematical vocabulary and symbols
CONTEXT **NCTM Element 2.c** Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts of mathematical problems.	The candidate includes all of the following elements: ● An example of a similar problem with a different context. ● An analysis of a similar problem (compare and contrast) ● An interpretation of the solution	The candidate includes two of the following elements: ● An example of a similar problem with a different context. ● An analysis of a similar problem (compare and contrast) ● An interpretation of the solution	The candidate includes one of the following elements: ● An example of a similar problem with a different context. ● An analysis of a similar problem (compare and contrast) ● An interpretation of the solution	The candidate does not include any of following elements: ● An example of a similar problem with a different context. ● An analysis of a similar problem (compare and contrast) ● An interpretation of the solution
NCTM PROCESS STANDARDS **NCTM Element 2.f**	The candidate includes a reflection on the process standards that includes a	The candidate includes a reflection on the process standards that includes a	The candidate includes a reflection on the process standards that includes a	The candidate includes a reflection on the process standards that includes a

Use and assist teachers in using resources from professional mathematics education organizations such as teacher/leader discussion groups, teacher networks, and print, digital, and virtual resources/ collections	description of how each of the five NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using all of the following elements: ● Teacher/Leader discussion groups ● Teacher networks ● Print, digital, and virtual resources/ collections	description of how four of the five NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using two of the following elements: ● Teacher/Leader discussion groups ● Teacher networks ● Print, digital, and virtual resources/ collections	description of how three of the five NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using one of the following elements: ● Teacher/Leader discussion groups ● Teacher networks ● Print, digital, and virtual resources/ collections	description of how one or two NCTM Process Standards impact the mathematical understanding. The reflection does not include any specific instances where the candidate assisted teachers using all of the following elements: ● Teacher/Leader discussion groups ● Teacher networks ● Print, digital, and virtual resources/ collections

Numbers Systems and Number Theory Reflection 3:
Analyzing Math Games
Course Performance Based Assessment

This is a Performance Based Assessment that addresses the following NCTM Math Specialist Standards:

- NCTM Elements: 1a, 2a, 2b, 2c, 2f
- NCTM Content Standard: C.1.3
    - Fundamental ideas of number theory – divisors, factors and factorization, multiples, primes and composite numbers.
- Focus of Reflection: Analyzing and aligning math games to math standards and diverse learners

The purpose of this Course Performance Based Assessment is for the candidate to demonstrate preparedness to support the development of student mathematical proficiency. All elementary mathematics specialists should know the above topics related to numbers and operations with their content understanding and mathematical practices.

The candidate will complete an activity and written reflection connected to their current mathematical understanding and how it has changed over the duration of the course. The final product will be submitted on Blackboard in Tk20. For a complete rubric and grading criteria please see the rubric below.

Summary of Actions

- Complete the game activity.
- As a group, reflect on one game by answering the questions for the written reflection paper.
- Each student will submit the following to Tk20: 1) game, work samples, and/or photos; and 2) written reflection paper.

WRITTEN REFLECTION PAPER

The written reflection paper should be clearly written so that the candidate's understanding of the involved numbers systems and number theory content and mathematical practices is evident. There are five guiding questions with sub-questions that should be addressed in the written reflections:

1. **What conceptual or procedural knowledge was gained by engaging in your collection of tasks/games? Describe how the task/game uses strategy to develop the conceptual and procedural knowledge (If it doesn't, describe how you would modify it so that it does)?**
2. **How does the use of the task/game promote problem solving, multiple representations, generalizations, connections, and assist the formulation of student generalizations (If it doesn't, describe how you would modify it so that it does)?**

3. **How does this task/game support learners to use mathematical modeling, appropriate mathematical vocabulary, and symbols during task/game play (If it doesn't, describe how you would modify it so that it does)?**
4. **Create or cite an open-ended (multiple ways of solving), multiple-solution (multiple correct solutions), contextual (real world) task that is aligned to the mathematical standards of the task/game. In what ways does the game support the mathematical understandings of this task?**
5. **Which of the five NCTM Process Standards (Problem Solving, Reasoning & Proof, Representations, Connections & Communications) was least evident in the task/game. Describe a modification to the game that would enhance this standard.**

Analyzing and aligning games to mathematical standards and diverse learners

NCTM Elements: 1a, 2a, 2b, 2c, 2f

Analyzing Math Games & Groupworthy Tasks
NCTM Content Standard: C.1.2 Fundamental ideas of number theory – divisors, factors and factorization, multiples, primes and composite numbers.
Essential Understanding: Children develop number theory through a variety of experiences, including games. Games provide an opportunity for students to increase arithmetic fluency while using strategy to analyze patterns and number theory. Mistakes and misconceptions are essential in the development of game strategy and number sense. • What mistakes lead to strategies and recognition of number patterns? • How aligned is a game to the mathematical standards? • How does a math game meet the needs of diverse learners?

Analyzing Math Games & Groupworthy tasks

Phase 1: Explore and analyze a variety of math games and groupworthy tasks.
As a mathematics specialist, you will need to analyze and evaluate games and activities in order to determine if they meet the instructional goal and the needs of diverse learners. In phase 1, you will explore and analyze a variety of math games. You will use supplemental readings and group collaboration to determine the quality of the games.

Phase 2: Identify four math games and collaborate with your assigned group to determine the value of the games.
Each group member will identify four quality math games that meet the following criteria:
Game 1 - Uses technology

Game 2 - Published by: John Van de Walle, Burns, Fosnot, Illuminations, Math Solutions, Kathy Richardson or other as approved by the instructor

Game 3 - Uses an ELL/SIOP strategy or sentence frame

Game 4 - Uses minimal materials and can be sent home with a child at zero or minimal cost

Phase 3: Game Day

Groups will present their games during a game day. This is not a lecture presentation. All directions must be available for playing without assistance.

Phase 4: Group write-up

Groups will write up the reflection together, and each candidate will submit the reflection to TK-20 individually.

Numbers Systems and Number Theory Reflection
Course Performance Based Assessment

Level/Criteria	4	3	2	1
	Exceeds Expectations	**Meets Expectations**	**Developing**	**Does Not Meet Expectations**
BUILDING CONCEPTUAL AND PROCEDURAL UNDERSTANDING **NCTM Element 1.a** Demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, applications in varied contexts and connections.	The candidate includes all of the following elements: ● Application of conceptual and procedural knowledge in identifying solutions in the problem set ● Explanation of the development of conceptual to procedural knowledge ● Discussion of new knowledge gained and the connections to past knowledge and experiences	The candidate includes two of the following elements: ● Application of conceptual and procedural knowledge in identifying solutions in the problem set ● Explanation of the development of conceptual to procedural knowledge ● Discussion of new knowledge gained and the connections to past knowledge and experiences	The candidate includes one of the following elements: ● Application of conceptual and procedural knowledge in identifying solutions in the problem set ● Explanation of the development of conceptual to procedural knowledge ● Discussion of new knowledge gained and the connections to past knowledge and experiences	The candidate does not include any of following elements: ● Application of conceptual and procedural knowledge in identifying solutions in the problem set ● Explanation of the development of conceptual to procedural knowledge ● Discussion of new knowledge gained and the connections to past knowledge and experiences
PROBLEM SOLVING **NCTM Element 2.a** Use problem solving to develop conceptual understanding, make a sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.	The candidate includes all of the following elements: ● Describes the use of problem solving within the problem set to formulate generalizations ● Explains how to make sense of the problems in the problem set ● Apply a variety of strategies and representations	The candidate includes two of the following elements: ● Use of problem solving within the problem set to formulate generalizations ● Make sense of the problems in the problem set ● Apply a variety of strategies and representations to the problem set	The candidate includes one of the following elements: ● Use of problem solving within the problem set to formulate generalizations ● Make sense of the problems in the problem set ● Apply a variety of strategies and representations to the problem set	The candidate does not include any of following elements: ● Use of problem solving within the problem set to formulate generalizations ● Make sense of the problems in the problem set ● Apply a variety of strategies and representations to the problem set

	to the problem set			
REPRESENTATIONS **NCTM Element 2.b** Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.	The candidate includes all of the following elements: ● Describes how multiple representations were used to model the problem set ● Discusses how the representations support the creation of generalizations ● Uses appropriate mathematical vocabulary and symbols	The candidate includes two of the following elements: ● Describes how multiple representations were used to model the problem set ● Discusses how the representations support the creation of generalizations ● Uses appropriate mathematical vocabulary and symbols	The candidate includes one of the following elements: ● Describes how multiple representations were used to model the problem set ● Discusses how the representations support the creation of generalizations ● Uses appropriate mathematical vocabulary and symbols	The candidate does not include any of following elements: ● Describes how multiple representations were used to model the problem set ● Discusses how the representations support the creation of generalizations ● Uses appropriate mathematical vocabulary and symbols
CONTEXT **NCTM Element 2.c** Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts of mathematical problems.	The candidate includes all of the following elements: ● An example of a similar problem with a different context. ● An analysis of a similar problem (compare and contrast) ● An interpretation of the solution	The candidate includes two of the following elements: ● An example of a similar problem with a different context. ● An analysis of a similar problem (compare and contrast) ● An interpretation of the solution	The candidate includes one of the following elements: ● An example of a similar problem with a different context. ● An analysis of a similar problem (compare and contrast) ● An interpretation of the solution	The candidate does not include any of following elements: ● An example of a similar problem with a different context. ● An analysis of a similar problem (compare and contrast) ● An interpretation of the solution
NCTM PROCESS STANDARDS **NCTM Element 2.f**	The candidate includes a reflection on the process standards that includes a	The candidate includes a reflection on the process standards that includes a	The candidate includes a reflection on the process standards that includes a	The candidate includes a reflection on the process standards that includes a

Use and assist teachers in using resources from professional mathematics education organizations such as teacher/leader discussion groups, teacher networks, and print, digital, and virtual resources/ collections	description of how each of the five NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using all of the following elements: ● Teacher/Leader discussion groups ● Teacher networks ● Print, digital, and virtual resources/ collections	description of how four of the five NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using two of the following elements: ● Teacher/Leader discussion groups ● Teacher networks ● Print, digital, and virtual resources/ collections	description of how three of the five NCTM Process Standards impact the mathematical understanding. The reflection includes specific instances where the candidate assisted teachers using one of the following elements: ● Teacher/Leader discussion groups ● Teacher networks ● Print, digital, and virtual resources/ collections	description of how one or two NCTM Process Standards impact the mathematical understanding. The reflection does not include any specific instances where the candidate assisted teachers using all of the following elements: ● Teacher/Leader discussion groups ● Teacher networks ● Print, digital, and virtual resources/ collections

Reflection Log 5 rubric

Course Performance Based Assessment

Candidates will explore the historical development and perspectives of number, operations, number systems, and quantity including contributions of significant figures and diverse cultures. Candidates will identify a non-Eurocentric mathematician who contributed to the key ideas of number systems and number theory, and present a 5-minute Ignite Talk on this mathematician in class. The final product(s) will be submitted on Blackboard in Tk20.

Levels/Criteria	4	3	2	1
	Exceeds Expectations	**Meets Expectations**	**Developing**	**Does Not Meet Expectations**
NCTM Indicator C.1.5 Historical development of numbers and operations.	Essay describes the historical development of numbers and operations in depth and provides specific examples.	Essay describes the historical development of numbers and operations and provides specific examples.	Essay describes the historical development of numbers and operations and provides an example.	Essay includes incomplete description of historical development of numbers and operations.
NCTM Indicator C.1.5 Historical perspectives of numbers and operations.	Essay describes the historical perspectives of numbers and operations in depth and provides specific examples.	Essay describes the historical perspectives of numbers and operations and provides specific examples.	Essay describes the historical perspectives of numbers and operations and provides an example.	Essay includes incomplete description of historical perspectives of numbers and operations.
NCTM Indicator C.1.5 Contributions of historically significant figures and diverse cultures.	Essay describes contributions of historically significant figures and diverse cultures in depth and provides specific examples.	Essay describes contributions of historically significant figures and diverse cultures and provides specific examples.	Essay describes contributions of historically significant figures and diverse cultures and provides an example.	Essay includes incomplete description of historically significant figures and diverse cultures.