George Mason University
College of Education and Human Development
Graduate School of Education
Mathematics Education Leadership

MATH 614.6M3– Rational Numbers and Proportional Reasoning for K-8 Teachers
3 Credits, Spring 2019
Monday 7:20PM – 10:00PM; Online

Faculty

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Prerequisites/Corequisites

Admission to the Mathematics Education Leadership Master's Degree Program or instructor permission.

University Catalog Course Description

This course will cover the basic number strands in fractions and rational numbers, decimals and percent, and ratios and proportions in the school curriculum. Instruction will cover interpretations, computations, and estimation with a coordinated program of activities that develop both rational number concepts and skills and proportional reasoning. Offered by Mathematics. May not be repeated for credit.

Course Overview

This course, for future K-8 mathematics teacher specialists, examines concepts contained in the rational number strands of the Virginia Standards of Learning (SOL), Common Core State Standards (CCSS), and/or referenced in the National Council of Teachers of Mathematics (NCTM) Principles and Standards. Through a coordinated program of activities, participants will learn to explore the structure of rational number systems, properties of numbers and develop number sense, computation and estimation concepts and skills.

Course Delivery Method

This course will be delivered online (76% or more) using a synchronous format via Blackboard Learning Management system (LMS) housed in the MyMason portal. You will log in to the

Blackboard (Bb) course site using your Mason email name (everything before @masonlive.gmu.edu) and email password. The course site will be available on January 10, 2019.

Under no circumstances, may candidates/students participate in online class sessions (either by phone or Internet) while operating motor vehicles. Further, as expected in a face-to-face class meeting, such online participation requires undivided attention to course content and communication.

Technical Requirements

To participate in this course, students will need to satisfy the following technical requirements:

- High-speed Internet access with standard up-to-date browsers. To get a list of Blackboard's supported browsers see: https://help.blackboard.com/Learn/Student/Getting_Started/Browser_Support#supported-browsers

  To get a list of supported operation systems on different devices see: https://help.blackboard.com/Learn/Student/Getting_Started/Browser_Support#tested-devices-and-operating-systems
- Students must maintain consistent and reliable access to their GMU email and Blackboard, as these are the official methods of communication for this course.
- Students will need a headset microphone for use with the Blackboard Collaborate web conferencing tool. [Delete this sentence if not applicable.]
- Students may be asked to create logins and passwords on supplemental websites and/or to download trial software to their computer or tablet as part of course requirements.
- The following software plug-ins for PCs and Macs, respectively, are available for free download: [Add or delete options, as desire.]
  - Adobe Acrobat Reader: https://get.adobe.com/reader/
  - Windows Media Player: https://support.microsoft.com/en-us/help/14209/get-windows-media-player
  - Apple Quick Time Player: www.apple.com/quicktime/download/

Expectations

- Course Week: Our course week will begin on the day that our synchronous meetings take place as indicated on the Schedule of Classes.
- Log-in Frequency:
  Students must actively check the course Blackboard site and their GMU email for communications from the instructor, class discussions, and/or access to course materials at least 2 times per week. In addition, students must log-in for all scheduled online synchronous meetings.
- Participation:

Students are expected to actively engage in all course activities throughout the semester, which includes viewing all course materials, completing course activities and assignments, and participating in course discussions and group interactions.

- Technical Competence:
  Students are expected to demonstrate competence in the use of all course technology. Students who are struggling with technical components of the course are expected to seek assistance from the instructor and/or College or University technical services.
- Technical Issues:
  Students should anticipate some technical difficulties during the semester and should, therefore, budget their time accordingly. Late work will not be accepted based on individual technical issues.
- Workload:
  Please be aware that this course is **not** self-paced. Students are expected to meet *specific deadlines* and *due dates* listed in the **Class Schedule** section of this syllabus. It is the student's responsibility to keep track of the weekly course schedule of topics, readings, activities and assignments due.
- Instructor Support:
  Students may schedule a one-on-one meeting to discuss course requirements, content or other course-related issues. Those unable to come to a Mason campus can meet with the instructor via telephone or web conference. Students should email the instructor to schedule a one-on-one session, including their preferred meeting method and suggested dates/times.
- Netiquette:
  The course environment is a collaborative space. Experience shows that even an innocent remark typed in the online environment can be misconstrued. Students must always re-read their responses carefully before posting them, so as others do not consider them as personal offenses. *Be positive in your approach with others and diplomatic in selecting your words*. Remember that you are not competing with classmates, but sharing information and learning from others. All faculty are similarly expected to be respectful in all communications.
- Accommodations:
  Online learners who require effective accommodations to insure accessibility must be registered with George Mason University Disability Services.

Learner Outcomes or Objectives

This course is designed to enable students to do the following:

1. Use numerous representations and conceptual models
2. Develop flexibility in problem solving
3. Explain number concepts and interpret student work in many ways

Professional Standards

Standard 1: Content Standards

To be prepared to support the development of student mathematical proficiency, all elementary mathematics specialists should know the following topics related to number and operations with their content understanding and mathematical practices supported by appropriate technology and varied representational tools, including concrete models:

C.1.1 Counting and cardinality, comparing and ordering, understanding the structure of the base ten number system with particular attention to place value, order of magnitude, one-to-one correspondence, properties, and relationships in numbers and number systems – whole numbers, integers, **rationals, irrationals and reals**.

C.1.2 Arithmetic operations (addition, subtraction, multiplication, and division) including mental mathematics and standard and non-standard algorithms, interpretations, and representations of numbers – fractions, decimals, rationals, irrationals and reals.

C.1.4 Quantitative reasoning and relationships that include ratio, rate, proportion, and the use of units in problem situations

Standard 2: Mathematical Practices (NCTM NCATE Mathematics Content for Elementary Mathematics Specialist *Addendum to the NCTM NCATE Standards 2012)*
Effective elementary mathematics specialists solve problems, represent mathematical ideas, reason, prove, use mathematical models, attend to precision, identify elements of structure, generalize, engage in mathematical communication, and make connections as essential mathematical practices. They understand that these practices intersect with mathematical content and that understanding relies on the ability to demonstrate these practices within and among mathematical domains and in their teaching and mathematics leadership.

In their role as teacher, lead teacher, and/or coach/mentor, elementary mathematics specialist candidates:

2a) Use problem solving to develop conceptual understanding, make sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.

2b) Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.

2c) Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts or mathematical problems.

2f) Model how the development of mathematical understanding within and among mathematical domains intersects with the mathematical practices of problem solving, reasoning, communicating, connecting, and representing.

Required Texts

Empson, S. B. & Levi, L. (2011). *Extending children's mathematics: Fractions and decimals.* Portsmouth, NH: Heinemann.

Lamon, S.J. (2012). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers* (3rd edition). New York, NY: Routledge.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: NCTM.

Suggested Texts

Van de Walle, J., Karp, K, & Bay-Williams, J. (2018). *Elementary and middle school mathematics: Teaching developmentally* (10th edition). Boston, MA: Pearson Education.

Course Performance Evaluation

Students are expected to submit all assignments on time in the manner outlined by the instructor (e.g., Blackboard, Tk20, hard copy).

1. **PARTICIPATION (20%)**
   A commitment to participation in class discussions and course activities depends heavily and primarily on the regular attendance and participation of all involved. Participation will include taking part in discussions informed by critical reading and thinking, leading discussions about selected mathematics problems, and sharing with the class the products of various writing, reflection, lesson planning, and field experience assignments. The expectations, demands and workload of this course are professional and high. This requires students to consider number systems and number theory using different strategies and a variety of manipulatives and resources. During math work time, students should be developing strategies and non-traditional algorithms for the entire work time or discussing and sharing algorithms with each other. During math-talk and discussion times, students should be actively engaged by voicing their thoughts and connecting to topics presented during the discussion.

   Participation in this course requires a commitment to reading reflectively and critically the assigned readings. The readings will be used to provide a framework and coherent theme to the course content. They have been selected to introduce themes in professional development as well as research and critical commentary on current issues in mathematics education.

   Additional information regarding participation, tardies and absences can be found on Blackboard.

2. **RATIONAL NUMBERS REFLECTIONS - 3 (30%)**
   (*NCTM NCATE* 1.1, 2a, 2b, 2c, 2f)
   The purpose of this Course Performance Based Assessment is for the candidate to demonstrate preparedness to support the development of student mathematical proficiency.

All elementary mathematics specialists should know the identified content topics and the mathematical practices that can be used to develop those understandings.

These **three** assignments require a written reflection connected to the candidate's current mathematical understanding and how it has changed. The final products will be submitted on Blackboard in Tk20. For a complete project description, rubric and grading criteria please see assignment descriptions at the end of the syllabus and/or on Blackboard.

3. **CONTENT ASSESSMENTS (30%)**
   This course will explore various problems that require candidates to consider mathematics using a variety of representations. Complete assessment descriptions and grading criteria can be found on Blackboard.

4. **WEEKLY HOMEWORK ASSIGNMENTS (20%)**
   To further candidates' exploration of mathematics content, homework assignments are assigned weekly. Candidates should bring assignments to class and be prepared to share strategies and misconceptions.

5. **Other Requirements**
   It is your responsibility to attend all class sessions. Please report your reasons for any absences to the instructor in writing.

   Tardiness: It is your responsibility to be on time for each class session. Please report your reasons for any tardiness to the instructor in writing.

   Class materials will be posted for each class session on Blackboard. Students are responsible for reviewing these materials and submitting required artifacts (where appropriate) to online class discussion boards.

   All assignments are to be turned in to your instructor on time. **Late work will not be accepted for full credit.** Assignments turned in late will receive a 10% deduction from the grade per late day or any fraction thereof (including weekends and holidays).

- **Grading**

A 93%-100%	B+ 87%-89%	C 70%-79%
A- 90%-92%	B 80%-86%	F Below 70%

For Master's Degrees:
Candidates must have a minimum GPA of 3.00 in coursework presented on the degree application, which may include no more than 6 credits of C. (Grades of C+, C-, or D do not apply to graduate courses. The GPA calculation excludes all transfer courses and Mason non-degree studies credits not formally approved for the degree).

For Endorsement Requirements
Candidates must have a grade of B or higher for all licensure coursework (endorsement coursework).

Professional Dispositions

See https://cehd.gmu.edu/students/polices-procedures/

Class Schedule

Date	Topic(s)	Readings Due	Due
Week 1 01/28	Syllabus Overview *Principles to Actions* (NCTM, 2014): The Mathematics Teaching Practices Connecting to the TRU Framework Proportional Reasoning: An overview Pre-Assessment		
Week 2 02/04	Proportional Reasoning: An overview Access and Equity	Empson & Levi: Foreword, Introduction, Chapter 1-2 Lamon: Chapter 1 PTA: p. 59-69	Homework 1
Week 3 02/11	**ASYNCHRONOUS** Fractions and Rational Numbers Implement tasks that promote reasoning and problem solving	Empson & Levi: Chapter 3 Lamon: Chapter 2 PTA: p. 17-24	Introduction Assignment: Post Part 1 to Discussion Board, Part 2 to Assignments Content Assessment 1
Week 4 02/18	Relative Thinking Pose purposeful questions	Empson & Levi: Chapter 4-5 Lamon: Chapter 3 PTA: p. 35 - 42	Homework 2
Week 5 02/25	Relative Thinking Establish mathematics goals to focus learning	Lamon: Chapter 4 PTA: p. 12-17	**Reflection 1 (Uploaded to TK20)** Homework 3
Week 6 03/04	Fractions Curriculum	Lamon: Chapter 5 PTA: p. 70-78	Homework 4

Week 7 03/11	Reasoning with Fractions Facilitate meaningful mathematical discourse	Empson & Levi: Chapter 6 Lamon: Chapter 6 PTA: p. 29-35	Homework 5
Week 8 03/18	Reasoning with Fractions Elicit and use evidence of student thinking	Lamon: Chapter 7 PTA: p. 53-58	**Reflection 2 (Uploaded to TK20)** Homework 6
Week 9 03/25	Fractions as Part-Whole Comparisons Use and connect mathematical representations	Empson & Levi: Chapter 7 Lamon: Chapter 8 (first half) PTA: p. 24-29	Homework 7 Content Assessment 2
Week 10 04/01	Fractions as Part-Whole Comparisons Build procedural fluency from conceptual understanding	Lamon: Chapter 8 (second half) PTA: p. 42-48	Homework 8
Week 11 04/08	Fractions as Quotients Support productive struggle in learning mathematics	Empson & Levi: Chapter 8 Lamon: Chapter 9 (first half) PTA: p. 48-53	Homework 9
Week 12 04/15	No Class; Spring Break		
Week 13 04/22	Fractions as Operators Tools and technology	Lamon: Chapter 9 (second half) PTA: p. 78-89	**Reflection 3 (Uploaded to TK20)** Homework 10
Week 14 04/29	Fractions as Measures Assessment	Lamon: Chapter 10 PTA: p. 89-99	Homework 11
Week 15 05/06	Ratios and Rates Professionalism	PTA: p. 99-118	Homework 12
Week 16 05/13	Changing Instruction Taking Action	Empson & Levi: Chapter 9 Lamon: Chapter 11	Homework 13 Content Assessment 3

Note: Faculty reserves the right to alter the schedule as necessary, with notification to students.

Core Values Commitment

The College of Education and Human Development is committed to collaboration, ethical leadership, innovation, research-based practice, and social justice. Students are expected to adhere to these principles: http://cehd.gmu.edu/values/.

GMU Policies and Resources for Students

Policies

- Students must adhere to the guidelines of the Mason Honor Code (see https://catalog.gmu.edu/policies/honor-code-system/).
- Students must follow the university policy for Responsible Use of Computing (see http://universitypolicy.gmu.edu/policies/responsible-use-of-computing/).
- Students are responsible for the content of university communications sent to their Mason email account and are required to activate their account and check it regularly. All communication from the university, college, school, and program will be sent to students **solely** through their Mason email account.
- Students with disabilities who seek accommodations in a course must be registered with George Mason University Disability Services. Approved accommodations will begin at the time the written letter from Disability Services is received by the instructor (see https://ds.gmu.edu/).
- Students must silence all sound emitting devices during class unless otherwise authorized by the instructor.

Campus Resources

- Support for submission of assignments to Tk20 should be directed to tk20help@gmu.edu or https://cehd.gmu.edu/aero/tk20. Questions or concerns regarding use of Blackboard should be directed to http://coursessupport.gmu.edu/.
- For information on student support resources on campus, see https://ctfe.gmu.edu/teaching/student-support-resources-on-campus

For additional information on the College of Education and Human Development, please visit our website https://cehd.gmu.edu/students/ .

MATH 614 Rational Numbers Written Reflection Rubric

Rational Numbers Written Reflection (Course Performance-Based Assessment)				
Level/Criteria	**4**	**3**	**2**	**1**
	Exceeds Expectations	**Meets Expectations**	**Developing**	**Does Not Meet Expectations**
BUILDING CONCEPTUAL AND PROCEDURAL UNDERSTANDING NCTM Element 1a Demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, applications in varied contexts and connections within and among mathematical domains	The written reflection includes all of the following elements: · Applies conceptual and procedural knowledge in identifying solutions in the problem · Explains connections between conceptual and procedural knowledge · Discusses new knowledge in relation to past knowledge and experiences	The written reflection includes two of the following elements: · Applies conceptual and procedural knowledge in identifying solutions in the problem · Explains connections between conceptual and procedural knowledge · Discusses new knowledge in relation to past knowledge and experiences	The written reflection includes one of the following elements: · Applies conceptual and procedural knowledge in identifying solutions in the problem · Explains connections between conceptual and procedural knowledge · Discusses new knowledge in relation to past knowledge and experiences	The written reflection: · Does not apply conceptual and procedural knowledge in identifying solutions in the problem · Does not explain connections between conceptual and procedural knowledge · Does not discuss new knowledge in relation to past knowledge and experiences

PROBLEM SOLVING NCTM Element 2a Use problem solving to develop conceptual understanding, make a sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.	The written reflection includes all of the following elements: · Uses problem solving within the assigned problem to formulate generalizations · Makes sense of the problems within the assigned problem · Applies a variety of strategies and representations within the assigned problem	The written reflection includes two of the following elements: · Uses problem solving within the assigned problem to formulate generalizations · Makes sense of the problems within the assigned problem · Applies a variety of strategies and representations within the assigned problem to the assigned problem	The written reflection includes one of the following elements: · Uses problem solving within the assigned problem to formulate generalizations · Makes sense of the problems within the assigned problem · Applies a variety of strategies and representations within the assigned problem to the assigned problem	The written reflection: · Does not use problem solving within the assigned problem to formulate generalizations · Does not make sense of the problems within the assigned problem · Does not apply a variety of strategies and representations within the assigned problem

REPRESENTATIONS NCTM Element 2b Represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.	The written reflection includes all of the following elements: · Describes how multiple representations were used to model the problem · Discusses how the representations support the creation of generalizations · Uses appropriate mathematical vocabulary and symbols	The written reflection includes two of the following elements: · Describes how multiple representations were used to model the problem · Discusses how the representations support the creation of generalizations · Uses appropriate mathematical vocabulary and symbols	The written reflection includes one of the following elements: · Describes how multiple representations were used to model the problem · Discusses how the representations support the creation of generalizations · Uses appropriate mathematical vocabulary and symbols	The written reflection: · Does not describe how multiple representations were used to model the problem · Does not discuss how the representations support the creation of generalizations · Does not use appropriate mathematical vocabulary and symbols
CONTEXT NCTM Element 2c Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts of mathematical problems.	The reflection includes all of the following elements: · An example of a similar problem with a different context. · An analysis of a similar problem (compare and contrast) · An interpretation of the solution	The reflection includes two of the following elements: · An example of a similar problem with a different context. · An analysis of a similar problem (compare and contrast) · An interpretation of the solution	The reflection includes one of the following elements: · An example of a similar problem with a different context. · An analysis of a similar problem (compare and contrast) · An interpretation of the solution	The reflection does not include the following elements: · An example of a similar problem with a different context. · An analysis of a similar problem (compare and contrast) · An interpretation of the solution

NCTM PROCESS STANDARDS NCTM Element 2f Model how the development of mathematical understanding within and among mathematical domains intersects with the mathematical practices of problem solving, reasoning, communicating, connecting, and representing.	The reflection includes a description of how each of the five NCTM Process Standards impact the mathematical understanding. Process Standards are: · problem solving · reasoning · communicating · connecting · representing	The reflection includes a description of how four of the five NCTM Process Standards impact the mathematical understanding. Process Standards are: · problem solving · reasoning · communicating · connecting · representing	The reflection includes a description of how three of the five NCTM Process Standards impact the mathematical understanding. Process Standards are: · problem solving · reasoning · communicating · connecting · representing	The reflection includes a description of how two or less of the five NCTM Process Standards impact the mathematical understanding. Process Standards are: · problem solving · reasoning · communicating · connecting · representing