George Mason University
College of Education and Human Development
Mathematics Education Leadership

MATH 611.B01 – Geometry and Measurement for K-8 Teachers
3 Credits, Summer 2018
June 4 – July 28 Mondays & Wednesdays (4:30-7:10) Online

Faculty

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Prerequisites/Corequisites

Admission to the Mathematics Education Leadership Master's Degree Program or instructor permission. Enrollment limited to students with a class of Advanced to Candidacy, Graduate, Non-Degree or Senior Plus. Enrollment is limited to Graduate, Non-Degree or Undergraduate level students. Students in a Non-Degree Undergraduate degree may **not** enroll.

University Catalog Course Description

The course explores the foundations of informal measurement and geometry in one, two, and three dimensions. The van Hiele model for geometric learning is used as a framework for how children build their understanding of length, area, volume, angles, and geometric relationships. Visualization, spatial reasoning, and geometric modeling are stressed. As appropriate, transformational geometry, congruence, similarity, and geometric constructions will be discussed.

Course Overview

This course is for future K-8 mathematics teacher specialists will cover the NCTM, Common Core State Standards and Virginia SOL strands in geometry and measurement, especially those in grades 5-8. Special attention will be given to interpreting and assessing students' work and learning.

Course Delivery Method

This course will be delivered online (76% or more) using a synchronous format via Blackboard Learning Management system (LMS) housed in the MyMason portal. You will log in to the

Blackboard (Bb) course site using your Mason email name (everything before @masonlive.gmu.edu) and email password. The course site will be available on May 9, 2017.

Under no circumstances, may candidates/students participate in online class sessions (either by phone or Internet) while operating motor vehicles. Further, as expected in a face-to-face class meeting, such online participation requires undivided attention to course content and communication.

Technical Requirements

To participate in this course, students will need to satisfy the following technical requirements:

- High-speed Internet access with a standard up-to-date browser, either Internet Explorer or Mozilla Firefox is required (note: Opera and Safari are not compatible with Blackboard).
- Students must maintain consistent and reliable access to their GMU email and Blackboard, as these are the official methods of communication for this course.
- Students will need a headset microphone for use with the Blackboard Collaborate web conferencing tool.
- Students will need video tools such as a web camera.
- Students may be asked to create logins and passwords on supplemental websites and/or to download trial software to their computer or tablet as part of course requirements.
- The following software plug-ins for PCs and Macs, respectively, are available for free download:
    - Adobe Acrobat Reader: https://get.adobe.com/reader/
    - Windows Media Player: https://windows.microsoft.com/en-us/windows/downloads/windows-media-player/
    - Apple Quick Time Player: www.apple.com/quicktime/download/

Expectations

- Course Week: Our course week will begin on the day that our synchronous meetings take place as indicated on the Schedule of Classes.
- Log-in Frequency:
  Students must actively check the course Blackboard site and their GMU email for communications from the instructor, class discussions, and/or access to course materials at least 2 times per week. In addition, students must log-in for all scheduled online synchronous meetings.
- Participation:
  Students are expected to actively engage in all course activities throughout the semester, which includes viewing all course materials, completing course activities and assignments, and participating in course discussions and group interactions.
- Technical Competence:
  Students are expected to demonstrate competence in the use of all course technology. Students who are struggling with technical components of the course are expected to seek assistance from the instructor and/or College or University technical services.

- Technical Issues:
  Students should anticipate some technical difficulties during the semester and should, therefore, budget their time accordingly. Late work will not be accepted based on individual technical issues.
- Workload:
  Please be aware that this course is **not** self-paced. Students are expected to meet *specific deadlines* and *due dates* listed in the **Class Schedule** section of this syllabus. It is the student's responsibility to keep track of the weekly course schedule of topics, readings, activities and assignments due.
- Instructor Support:
  Students may schedule a one-on-one meeting to discuss course requirements, content or other course-related issues. Those unable to come to a Mason campus can meet with the instructor via telephone or web conference. Students should email the instructor to schedule a one-on-one session, including their preferred meeting method and suggested dates/times.
- Netiquette:
  The course environment is a collaborative space. Experience shows that even an innocent remark typed in the online environment can be misconstrued. Students must always re-read their responses carefully before posting them, so as others do not consider them as personal offenses. *Be positive in your approach with others and diplomatic in selecting your words*. Remember that you are not competing with classmates, but sharing information and learning from others. All faculty are similarly expected to be respectful in all communications.
- Accommodations:
  Online learners who require effective accommodations to insure accessibility must be registered with George Mason University Disability Services.

Learner Outcomes or Objectives

This course is designed to enable students to do the following:

1. Candidates will develop a comprehensive understanding of an axiomatic system of reasoning, representation and creation of shapes using old and new ways to draw and construct the concepts of measurement and symmetry of structure.
2. Candidates will examine in depth geometry content appropriate for K-8 mathematics teachers, including the use of technology to study geometry and historical connections to geometry.
3. Candidates will explore fundamentals of geometry, congruence and similarity, two-and three- dimensional figures, transformations and isometries, and basic measurement properties of perimeter, area, and volume.
4. Candidates will examine two frameworks, Van Hiele's model for geometric learning and geometric Habits of Mind, in order to assess their own progress throughout the course and to discover these models' pedagogical implications on classroom instruction.

Professional Standards (National Council of Teachers of Mathematics)

Upon completion of this course, students will have met the following professional standards:

Geometry and Measurement (NCTM NCATE Mathematics Content for Elementary Mathematics Specialist *Addendum to the NCTM NCATE Standards 2012)*

Standard 1: Content Knowledge (NCTM NCATE Mathematics Content for Elementary Mathematics Specialist *Addendum to the NCTM NCATE Standards 2012)* Effective elementary mathematics specialists demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, connections, and applications within and among mathematical content domains. Elementary mathematics specialist candidates: 1a) Demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, applications in varied contexts, and connections within and among mathematical domains (Number and Operations, Algebra, Geometry and Measurement, and Statistics and Probability) as outlined in the NCTM CAEP Mathematics Content for Elementary Mathematics Specialist.

To be prepared to support the development of student mathematical proficiency, all elementary mathematics specialists should know the following topics related to geometry and measurement with their content understanding and mathematical practices supported by appropriate technology and concrete models:

C.3.1 Core concepts including angle, parallel, and perpendicular, and principles of Euclidean geometry in two and three dimensions
C.3.2 Transformations including dilations, translations, rotations, reflections, glide reflections; compositions of transformations; and the expression of symmetry and regularity in terms of transformations
C.3.3 Congruence, similarity and scaling, and their development and expression in terms of transformations
C.3.4 Basic geometric figures in one, two, and three dimensions (line segments, lines, rays, circles, arcs, polygons, polyhedral solids, cylinders, cones, and spheres) and their elements (vertices, edges, and faces)
C.3.5 Identification, classification into categories, visualization, two- and three-dimensional representations, and formula rationale and derivation (perimeter, area, and volume) of two- and three-dimensional objects (triangles; classes of quadrilaterals such as rectangles, parallelograms, and trapezoids; regular polygons; rectangular prisms; pyramids; cones; cylinders; and spheres)
C.3.6 Geometric measurement and units (linear, area, surface area, volume, and angle), unit comparison, and the iteration, additivity, and invariance related to measurements
C.3.7 Geometric constructions, axiomatic reasoning, and making and proving conjectures about geometric shapes and relations
C.3.8 Coordinate geometry including the equations of lines and algebraic proofs (e.g., Pythagorean Theorem and its converse)
C.3.9 Historical development and perspectives of geometry and measurement including contributions of significant figures

Standard 2: Mathematical Practices (NCTM NCATE Mathematics Content for Elementary Mathematics Specialist *Addendum to the NCTM NCATE Standards 2012)* Effective elementary mathematics specialists solve problems, represent mathematical ideas, reason, prove, use mathematical models, attend to precision, identify elements of structure, generalize, engage in mathematical communication, and make connections as essential mathematical practices. They understand that these practices intersect with mathematical content and that understanding relies on the ability to demonstrate these practices within and among mathematical domains and in their

teaching and mathematics leadership. In their role as teacher, lead teacher, and/or coach/mentor, elementary mathematics specialist candidates:
2a) Use problem solving to develop conceptual understanding, make sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.
2b) Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.
2c) Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts or mathematical problems.
2f) Model how the development of mathematical understanding within and among mathematical domains intersects with the mathematical practices of problem solving, reasoning, communicating, connecting, and representing.

Standard 3: Content Pedagogy (NCTM NCATE Mathematics Content for Elementary Mathematics Specialist *Addendum to the NCTM NCATE Standards 2012)* Effective elementary mathematics specialists apply knowledge of curriculum standards for mathematics and their relationship to student learning within and across mathematical domains in teaching elementary students and coaching/mentoring elementary classroom teachers. They incorporate research-based mathematical experiences and include multiple instructional strategies and mathematics-specific technological tools in their teaching and coaching/mentoring to develop all students' mathematical understanding and proficiency. As teacher, lead teacher, and coach/mentor, they provide and assist teachers in providing students with opportunities to do mathematics – talking about it and connecting it to both theoretical and real-world contexts. They plan, select, implement, interpret, and assist teachers in the incorporation of formative and summative assessments for monitoring student learning, measuring student mathematical understanding, and informing practice. In their role as teacher, lead teacher, and/or coach/mentor, elementary mathematics specialist candidates:
3a) Apply knowledge of curriculum standards for elementary mathematics and their relationship to student learning within and across mathematical domains in teaching elementary students and coaching/mentoring elementary classroom teachers.
3c) Plan and assist others in planning lessons and units that incorporate a variety of strategies, differentiated instruction for diverse populations, and mathematics-specific and instructional technologies in building all students' conceptual understanding and procedural proficiency.
3e) Implement and promote techniques related to student engagement and communication including selecting high quality tasks, guiding mathematical discussions, identifying key mathematical ideas, identifying and addressing student misconceptions, and employing a range of questioning strategies

Standard 5: Impact on Student Learning (NCTM NCATE Mathematics Content for Elementary Mathematics Specialist *Addendum to the NCTM NCATE Standards 2012)* Elementary mathematics specialists provide evidence that as a result of their instruction or coaching/mentoring of teachers, elementary students' conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and application of major mathematics concepts in varied contexts have increased. Elementary mathematics specialists support the continual development of a positive disposition toward mathematics. These mathematics specialists show that new student mathematical knowledge

has been created as a consequence of their ability to engage students or coach/mentor teachers in mathematical experiences that are developmentally appropriate, require active engagement, and include mathematics-specific technology in building new knowledge. In their role as teacher, lead teacher, and/or coach/mentor, elementary mathematics specialist candidates:
5b) Engage students and coach/mentor teachers in using developmentally appropriate mathematical activities and investigations that require active engagement and include mathematics-specific technology in building new knowledge.

Required Texts

Schifter, D., Bastable, V., & Russell, S. J. (2017). *Examining Features of Shape: Casebook.* Reston, VA: NCTM.

Schifter, D., Bastable, V., & Russell, S. J. (2017). *Measuring Space in One, Two, and Three Dimensions: Casebook.* Reston, VA: NCTM.

Course Performance Evaluation

Students are expected to submit all assignments on time in the manner outlined by the instructor (e.g., Blackboard, Tk20, hard copy).

- **Assignments and/or Examinations**

  A. Professional Development Session (20%)
  NCATE/NCTM 5b
  Purpose: As a math leader, you will be responsible for supporting teachers, administrators and the school community in math understanding. In this class, you will prepare a professional development session that incorporates the following:
  - An interactive activity – not a lecture presentation
  - Includes math specific technology tool
  - References the reading

  B. Reflection Log (20%) – Performance Based Assessment
  NCATE/NCTM Indicator 1a (C.3.1 - C. 3.9), 2a, 2b, 2c, 2f
  Each class sessions will consist of working on a rich problem and utilizing the following: practices for promoting productive mathematics discussions, differentiation, the NCTM Process Standards and the multiple representations utilized. Students are expected to analyze and reflect solution strategies and come to class prepared to participate in the discussion.

  You will submit 9 reflection logs. Eight of the reflection logs should be approximately 1 page in length, include a picture or screenshot of student work, and answer the following questions:
  1) Reflect on the process of mathematical problem solving used in this rich task.
  2) What generalizations and conjectures did you make, and how did you use reasoning to construct viable arguments and proofs?
  3) How are different representations and models similar and how are they different?

4) How did you demonstrate flexibility in mathematical modeling when confronted with different models, representations, or solutions?
5) How could you see this problem in the real world?

C. Rich Task Project (20%) – Performance Based Assessment
NCATE/NCTM 3a, 3c, 3e, 5b
Develop a lesson that includes a rich task involving GEOMETRY. Plan should follow the template given and cover ideas and topics outlined in the rubric. Task must include some type of manipulative or technology usage. **See Rubric for additional details.**

D. Two Content Exams (40%)
NCATE/NCTM All Standards listed.
Students will take two content exams each worth 20% of the grade covering the content studied in the course. The focus of the exams will be on the mathematical content of the course. Students are expected to demonstrate their own understanding and reasoning of the content as well as the knowledge and understanding needed by K-8 students in order to make sense of this content.

- **Other Requirements**
  It is your responsibility to attend all class sessions. Please report your reasons for any absences to the instructor in writing.

  Tardiness: It is your responsibility to be on time for each class session. Please report your reasons for any tardiness to the instructor in writing.

  Class materials will be posted for each class session on Blackboard. Students are responsible for reviewing these materials and submitting required artifacts (where appropriate) to online class discussion boards.

  All assignments are to be turned in to your instructor on time. **Late work will not be accepted for full credit.** Assignments turned in late will receive a 10% deduction from the grade per late day or any fraction thereof (including weekends and holidays).

- **Grading**

A 93%-100%	B+ 87%-89%	C 70%-79%
A- 90%-92%	B 80%-86%	F Below 70%

For Master's Degrees:
Candidates must have a minimum GPA of 3.00 in coursework presented on the degree application, which may include no more than 6 credits of C. (Grades of C+, C-, or D do not apply to graduate courses. The GPA calculation excludes all transfer courses and Mason non-degree studies credits not formally approved for the degree).

For Endorsement Requirements
Candidates must have a grade of B or higher for all licensure coursework (endorsement

coursework).

Professional Dispositions

See https://cehd.gmu.edu/students/polices-procedures/

Students are expected to exhibit professional behaviors and dispositions at all times. Education professionals are held to high standards, both inside and outside of the classroom. Educators are evaluated on their behaviors and interactions with students, parents, other professionals, and the community at large. At the College of Education and Human Development, dispositions may play a part in the discussions and assignments of any/all courses in a student's program (and thus, as part or all of the grade for those assignments). For additional information visit: https://cehd.gmu.edu/students/polices-procedures/
In order to maintain a focused class, laptops and cell phones are to be used exclusively for the current class topic. Examples of this include searching for math standards, videos of mathematical algorithms, taking pictures of manipulatives, etc. Emailing, texting, and other forms of communication and social media are not permitted during class time unless it is directly related to the activity. In addition, students should refrain from grading papers and preparing lesson materials for their school placements during class time.

Class Schedule

Date	Topic	Readings and Assignments Due before start of class
Monday, June 4	Describing 2D and 3D objects	Readings - Examining Features of Shape - Chapter 1
Wednesday, June 6	Developing meaning for geometric terms Making sense of angle	Log 1 - Submit under ASSESSMENTS Readings - Examining Features of Shape - Chapters 2 & 3 Cut out triangles BEFORE class. (See blackboard)
Monday, June 11	Creating and applying definitions Comparing shapes	Log 2 - Submit under ASSESSMENTS Readings - Examining Features of Shape - Chapters 4 & 5
Wednesday, June 13	2-D images of a 3-D world Reasoning in geometric contexts	Log 3 - Submit under ASSESSMENTS Readings - Examining Features of Shape - Chapters 6 & 7

Monday, June 18	Different aspects of size	Log 4 - Submit under ASSESSMENTS PD 1 - Submit under DISCUSSION BOARD Readings - Measuring Space in One, Two, and Three Dimensions - Chapter 1
Wednesday, June 20	Composing and decomposing in one, two, and three dimensions	Log 5 - Submit under ASSESSMENTS PD 2 - Sumbit under DISCUSSION BOARD PBA: Task - Submit under ASSIGNMENTS Readings - Measuring Space in One, Two, and Three Dimensions - Chapter 2
Monday, June 25		Log 6 - Submit under ASSESSMENTS PD 3 - Sumbit under DISCUSSION BOARD PBA: Curriculum Standards and Research - Submit under ASSIGNMENTS Readings - Measuring Space in One, Two, and Three Dimensions - Chapter 3
Wednesday, June 27		Midterm
Monday, July 2	Rich tasks in geometry	(Due after class) PBA: Strategies and Misconceptions - Submit under ASSIGNMENTS
Wednesday, July 4	NO CLASS	NO CLASS
Monday, July 9	Measuring Length: What is a unit and how is it used?	PBA: Questions - Submit under ASSIGNMENTS
Wednesday, July 11	Measuring area: Structuring rectangles	Log 7 - Submit under ASSESSMENTS PD 4 - Submit under DISCUSSION BOARD PBA: Handout and Key - Submit under ASSIGNMENTS

		Readings - Measuring Space in One, Two, and Three Dimensions - Chapter 4
Monday, July 16	From rectangles to triangles and trapezoids	Log 8 - Submit under ASSESSMENTS PD 5 - Submit under DISCUSSION BOARD PBA: Technology - Submit under ASSIGNMENTS Readings - Measuring Space in One, Two, and Three Dimensions - Chapter 5
Wednesday, July 18	Measuring volume: Structuring boxes	PD 6 - Submit under DISCUSSION BOARD PBA: Differentiation - Submit under ASSIGNMENTS Readings - Measuring Space in One, Two, and Three Dimensions - Chapter 6
Monday, July 23	Same shape, different measures	***You will need a pre-packaged novelty ice cream cone and an orange for class*** PD 7 - Submit under DISCUSSION BOARD PBA: Reflections - Submit under ASSIGNMENTS Readings - Measuring Space in One, Two, and Three Dimensions - Chapter 7
Wednesday, July 25	Historical development of measurement and geometry	Log 9 - Historical Figure Log - Submit under ASSESSMENTS Final PBA - Submit under ASSESSMENTS Final Exam Due July 28 by 11:59pm

Note: Faculty reserves the right to alter the schedule as necessary, with notification to students.

Core Values Commitment

The College of Education and Human Development is committed to collaboration, ethical leadership, innovation, research-based practice, and social justice. Students are expected to adhere to these principles: http://cehd.gmu.edu/values/.

GMU Policies and Resources for Students

Policies

- Students must adhere to the guidelines of the Mason Honor Code (see http://oai.gmu.edu/the-mason-honor-code/).
- Students must follow the university policy for Responsible Use of Computing (see http://universitypolicy.gmu.edu/policies/responsible-use-of-computing/).
- Students are responsible for the content of university communications sent to their Mason email account and are required to activate their account and check it regularly. All communication from the university, college, school, and program will be sent to students **solely** through their Mason email account.
- Students with disabilities who seek accommodations in a course must be registered with George Mason University Disability Services. Approved accommodations will begin at the time the written letter from Disability Services is received by the instructor (see http://ods.gmu.edu/).
- Students must follow the university policy stating that all sound emitting devices shall be silenced during class unless otherwise authorized by the instructor.

Campus Resources

- Support for submission of assignments to Tk20 should be directed to tk20help@gmu.edu or https://cehd.gmu.edu/aero/tk20. Questions or concerns regarding use of Blackboard should be directed to http://coursessupport.gmu.edu/.
- The Writing Center provides a variety of resources and services (e.g., tutoring, workshops, writing guides, handbooks) intended to support students as they work to construct and share knowledge through writing (see http://writingcenter.gmu.edu/).
- The Counseling and Psychological Services (CAPS) staff consists of professional counseling and clinical psychologists, social workers, and counselors who offer a wide range of services (e.g., individual and group counseling, workshops and outreach programs)

to enhance students' personal experience and academic performance (see http://caps.gmu.edu/).

- The Student Support & Advocacy Center staff helps students develop and maintain healthy lifestyles through confidential one-on-one support as well as through interactive programs and resources. Some of the topics they address are healthy relationships, stress management, nutrition, sexual assault, drug and alcohol use, and sexual health (see http://ssac.gmu.edu/). Students in need of these services may contact the office by phone at 703-993-3686. Concerned students, faculty and staff may also make a referral to express concern for the safety or well-being of a Mason student or the community by going to http://ssac.gmu.edu/make-a-referral/.

For additional information on the College of Education and Human Development, please visit our website https://cehd.gmu.edu/.

Rich Task Project Rubric– Performance Based Assessment

Levels/Criteria	4	3	2	1
	Exceeds Expectations	**Meets Expectations**	**Developing**	**Does Not Meet Expectations**
a) CURRICULUM STANDARDS NCTM Indicator 3A.1 Apply knowledge of mathematics curriculum standards for elementary within and across mathematical domains.	Includes the grade level, major concept, objective/goals, VA SOL's, NCTM process standards, mathematical practice CCSS, and prerequisite knowledge. Cites research on the rationale for choosing a rich task and why your choice in task meets the requirements of a rich task.	Includes the grade level, major concept, objective/goals, VA SOL's, NCTM process standards, mathematical practice CCSS, and prerequisite knowledge. Lesson is based on research and it cited correctly.	Includes the grade level, major concept, objective/goals, VA SOL's, NCTM process standards, mathematical practice CCSS, and prerequisite knowledge.	Lesson plan is not based on research. Many aspects of the plan are missing.
3A.2 Relate mathematics curriculum standards to student learning.	Cites research on what elements make a "rich" question or a "high level" question. Cites research on what is a major mathematical concept or "big mathematical idea".			

b) COMPLETED LESSON PLAN 3C.5 Assist others in planning lessons and units that incorporate multiple strategies, differentiated instruction for diverse populations, and mathematics-specific and instructional technologies to build all students' conceptual understanding and procedural proficiency.	Lesson plan provides enough information and clarity for a teacher to implement the lesson and task as intended. Assisted peers in developing multiple strategies to build all students' conceptual understanding and procedural proficiency. Assisted peers in identifying diverse populations and modifying the task to build all students' conceptual understanding and procedural proficiency. Assisted peers in aligning mathematics-specific and instructional technologies to build all students' conceptual understanding and procedural proficiency.	Lesson plan provides enough information and clarity for a teacher to implement the lesson and task as intended with some questions. Assisted peers in developing multiple strategies **and** identifying diverse populations and modifying the task **or** aligning mathematics-specific and instructional technologies to build all students' conceptual understanding and procedural proficiency.	Lesson plan provides information for a teacher to implement the lesson and task as intended with many questions. Assisted peers in developing multiple strategies **or** identifying diverse populations and modifying the task **or** aligning mathematics-specific and instructional technologies to build all students' conceptual understanding and procedural proficiency.	Lesson plan is vague – teacher would be unable to implement the task as intended. Does not collaborate with peers on lesson plan.
c) QUESTIONS NCTM Indicator 3E.2 Provide instruction that incorporates high quality tasks and a range of questioning strategies. 3E.3 Guide productive mathematical discussions in classrooms centered on key mathematical ideas. 3E.4 Select and apply instructional techniques that assist in identifying and addressing student misconceptions.	Plan contains key questions and student expectations to aid teacher when implementing the task to maintain rigor. Plan contains questions and expected student expectations to address misconceptions. Plan contains questioning strategies to guide productive mathematical discussions in classrooms centered on key mathematical ideas. Questions engage students and teachers in communicating about mathematics. Plan describes how student misconceptions	Plan contains key questions and student expectations to aid teacher when implementing the task to maintain rigor but lacks some clarity. Plan contains questions and expected student expectations to address misconceptions. Questions engage students and teachers in communicating about mathematics.	Plan contains questions low in cognitive demand. Stude nt expectations are also low in rigor.	Plan does not contain key questions. Stude nt expectations are missing.

3E.5 Engage students and teachers in communicating about mathematics. 3E.6 Use students' misconceptions as opportunities for learning.	will be used as opportunities for learning.			
d) STRATEGIES & MISCONCEPTIONS NCTM Indicator 3C.1 Plan lessons and units that incorporate a variety of strategies. 3C.4 Build all students' conceptual understanding and procedural proficiency in planned lessons and units. 3C.6 Include in planned lessons and units multiple opportunities and solution avenues for students to demonstrate conceptual understanding and procedural proficiency.	Plan contains at least five different strategies that show multiple opportunities and solution avenues for students to demonstrate conceptual understanding and procedural proficiency. Plan contains at least two different misconceptions. Plan describes the connections between the different strategies/misconceptions using descriptions such as similarities, differences, efficiency, visual clarity, mathematical accuracy and/or precision to support students' conceptual understanding and procedural proficiency.	Plan contains four different strategies that show multiple opportunities and solution avenues for students to demonstrate conceptual understanding and procedural proficiency. Plan contains one misconception. Plan describes the connections between the different strategies/misconceptions to support students' conceptual understanding and procedural proficiency.	Plan contains three different strategies that show multiple opportunities and solution avenues for students to demonstrate conceptual understanding and procedural proficiency. Description of conceptual understanding and procedural proficiency is incomplete.	Student strategies and misconceptions lack a complete listing and in-depth understanding.
e1) TECHNOLOGY NCTM Indicator 3C.1 Include mathematics-specific and instructional technologies in planned lessons and units.	Your choice of technology is explained regarding how it is math-specific and supports the task. The tool is specific to the task (ie: the geoboard on NLVM, and not simply "iPads").	Your choice of technology is explained regarding how it is math-specific and supports the task. The tool is specific to the task (ie: the geoboard on NLVM, and not simply "iPads").	Your choice of technology is not explained regarding how it is math-specific and supports the task **or** the tool is not specific.	Your choice of technology is not explained regarding how it is math-specific and supports the task **and** the tool is not specific.

	Links to the web or appstore are provided and screen captures of the tool are included.			
e2) Developmental TECHNOLOGY NCTM Indicator 5B.3 Engage students in developmentally appropriate mathematical activities and investigations that include mathematics-specific technology in building new knowledge.	Your choice of technology is explained regarding how it will enhance learning. Tool engages students in developmentally appropriate mathematical activities and investigations that include mathematics-specific technology in building new knowledge. Plan contains a detailed explanation of how the students will interact with the tool.	Your choice of technology is explained regarding how it will enhance learning. Tool engages students in developmentally appropriate mathematical activities and investigations that include mathematics-specific technology in building new knowledge.	Your choice of technology is not explained regarding how it will enhance learning **or** the tool does not engage students in developmentally appropriate mathematical activities and investigations that include mathematics-specific technology in building new knowledge.	Your choice of technology is not explained regarding how it will enhance learning **and** the tool does not engage students in developmentally appropriate mathematical activities and investigations that include mathematics-specific technology in building new knowledge.
f1) DIFFERENTIATION NCTM Indicator 3A.3 Demonstrate how mathematics curriculum standards and learning progressions impact the teaching of elementary students at different developmental levels and coaching/mentoring elementary classroom teachers.	Plan identifies the grade level standard (VA SOL & CCSS) and at least two other grade levels and describes the progression and vertical alignment. Modifications to the lesson are given for different developmental levels to meet all student needs. Collaborated with peers to coach and give feedback on the differentiation of others' geometry task.	Plan includes how to modify the lesson to gear down and gear up to meet all student needs but is lacking clarity **or** completeness. Modifications are given that are appropriate for the given level but may lack clarity **or** completeness. Collaborated with peers to coach and give feedback on the differentiation of others' geometry task.	Plan includes how to modify the lesson to gear down and gear up to meet all student needs but is lacking clarity **and** completeness. Grade modifications are given that are appropriate for the given level but may lack clarity **and** completeness.	Plan differentiation and modifications are very minimal.
f2) DIVERSE POPULATIONS 3C.2 Plan lessons and units addressing student differences and diverse populations and how these differences influence student learning of mathematics.	Modifications to the lesson are given for diverse populations to meet all student needs. Explanation of how student differences may influence their learning of mathematics.	Modifications are given for diverse populations but may lack clarity **or** completeness.	Modifications are given for diverse populations but may lack clarity **and** completeness.	Plan modifications are very minimal.
g) HANDOUT & KEY <u>Geometry Task:</u> handout is clear and applicable	Geometry Task handout is clear and applicable for the grade	Geometry Task handout is applicable for the grade level but	Geometry Task handout is applicable for	Geometry Task handout is not applicable for the

for the grade level. Answer key is provided.	level. Answer key is provided.	lacks clarity. Answer key is provided.	the grade level but lacks clarity. Answer key is not provided.	grade level. Answer key is not provided.
h) TASK NCTM Indicator 3E.1 Implement and promote techniques for actively engaging students in learning and doing mathematics.	Geometry task implements and promotes techniques for actively engaging students in learning and doing mathematics.	Geometry Task has a cognitive demand of "Procedures with Connections"	Geometry Task has a cognitive demand of "Procedures without Connections"	Geometry Task has a cognitive demand of "Memorization"
i) LESSON REFLECTION - JUSTIFICATION NCTM Indicator 5B.1 Engage students in developmentally appropriate mathematical activities and investigations that require active engagement in building new knowledge.	Reflection shows evidence that students were engaged in developmentally appropriate mathematical activities and investigations that require active engagement in building new knowledge.	Reflection shows evidence that students were engaged in investigations that require active engagement in building new knowledge.	Reflection shows evidence that students were engaged in investigations that require active engagement.	Reflection of lesson implementation is missing.
j) LESSON REFLECTION - PROBLEM SOLVING 5B.4 Facilitate students' ability to develop future inquiries based on current analyses.	Reflection shows evidence that you facilitated students' ability to develop future inquiries based on current analyses.	Reflection shows evidence that you facilitated students' ability to develop inquiries about the task.	Reflection shows evidence that you facilitated students' ability solve the task.	There is no expectation of students communicating their problem solving strategies.
k) COACHING/ASSISTING NCTM INDICATOR 5B.2 Coach/mentor teachers in using developmentally appropriate mathematical activities and investigations that require active student engagement in building new knowledge.	Assist peers in designing a task that uses developmentally appropriate mathematical activities and investigations that require active student engagement in building new knowledge.	Assist peers in designing a task that is developmentally appropriate and uses investigations.	Assist peers in thinking about their task.	Does not collaborate with peers during task creation.

Reflection Log Rubric – Performance Based Assessment

Rubric for Logs 1-8

Log 1: C.3.1 Core concepts including angle, parallel, and perpendicular, and principles of Euclidean geometry in two and three dimensions

Log 2: C.3.2 Transformations including dilations, translations, rotations, reflections, glide reflections; compositions of transformations; and the expression of symmetry and regularity in terms of transformations

Log 3: C.3.3 Congruence, similarity and scaling, and their development and expression in terms of transformations

Log 4: C.3.4 Basic geometric figures in one, two, and three dimensions (line segments, lines, rays, circles, arcs, polygons, polyhedral solids, cylinders, cones, and spheres) and their elements (vertices, edges, and faces)

Log 5: C.3.5 Identification, classification into categories, visualization, two- and three-dimensional representations, and formula rationale and derivation (perimeter, area, and volume) of two- and three-dimensional objects (triangles; classes of quadrilaterals such as rectangles, parallelograms, and trapezoids; regular polygons; rectangular prisms; pyramids; cones; cylinders; and spheres)

Log 6: C.3.6 Geometric measurement and units (linear, area, surface area, volume, and angle), unit comparison, and the iteration, additivity, and invariance related to measurements

Log 7: C.3.7 Geometric constructions, axiomatic reasoning, and making and proving conjectures about geometric shapes and relations

Log 8: C.3.8 Coordinate geometry including the equations of lines and algebraic proofs (e.g., Pythagorean Theorem and its converse)

Criteria	Does not meet standard 1	Approaches standard 2	Meets standard 3	Exceeds standard 4
Section 1 – Rich Task Work Time				
NCTM CAEP Element 1A.1: Demonstrate knowledge of major concepts, algorithms, and procedures within and among mathematical domains.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 1A.2: Apply knowledge of major concepts, algorithms, procedures, applications in varied contexts, and connections within and among mathematical domains.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 1A.4: Apply conceptual and procedural knowledge of major concepts, algorithms, and	The candidate did not show evidence of the standard.	The candidate shows minimal	The candidate shows evidence of	The candidate shows evidence of

applications in building new knowledge from prior knowledge and experiences.		evidence of the standard.	mastery of the standard.	exceeding mastery of the standard.
NCTM CAEP Element 2B.2: Represent and model generalizations using mathematics.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 2B.3: Recognize structure and express regularity in patterns of mathematical reasoning.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 2B.4: Use multiple representations to model and describe mathematics.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 2B.5: Use appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
Section 2 – Rich Task Debate and Discussion				
NCTM CAEP Element 1A.3: Explain how concepts, algorithms, procedures, and applications have developed.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 2A.2: Make sense of a wide variety of problems and persevere in solving them.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 2A.3: Apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 2A.4: Formulate and test conjectures in order to frame generalizations.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 2B.1: Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.

NCTM CAEP Element 2B.6: Demonstrate an appreciation for mathematical rigor and inquiry.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 2F.1: Model how the development of mathematical understanding within and among mathematical domains intersects with the mathematical practices of problem solving, reasoning, communicating, connecting, and representing.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
Section 3 – Reflection Paper				
NCTM CAEP Element 2A.5: Monitor and reflect on the process of mathematical problem solving.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 2A.1: Use problem solving to develop conceptual understanding and to formulate and test generalizations.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 2F.2: Reflect on how the mathematical practices of problem solving, reasoning, communicating, connecting, and representing impact mathematical understanding.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 2C.2: Demonstrate flexibility in mathematical modeling when confronted with different purposes or contexts.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.
NCTM CAEP Element 2C.1: Formulate, represent, analyze, interpret, and validate mathematical models derived from real-world contexts or mathematical problems.	The candidate did not show evidence of the standard.	The candidate shows minimal evidence of the standard.	The candidate shows evidence of mastery of the standard.	The candidate shows evidence of exceeding mastery of the standard.

Reflection Log 9

The final reflection log will involve researching a major mathematical historical development and the contributions of a historically significant figure. We will discuss many of these developments and figures during the math talk all throughout the semester. However, this discussion will be brief. Once you find a topic that interests you, you should research it further. The following reflection should be about 2 pages in length and will be evaluated using the following criteria.

Levels/Criteria	**4**	**3**	**2**	**1**
	Exceeds Expectations	**Meets Expectations**	**Developing**	**Does Not Meet Expectations**
NCTM Indicator C.1.5 Historical development of number, operations, number systems, and quantity.	Essay describes the historical development of number, operations, number systems, and quantity in depth and provides specific examples.	Essay describes the historical development of number, operations, number systems, and quantity and provides specific examples.	Essay describes the historical development of number, operations, number systems, and quantity and provides an example.	Essay includes incomplete description of historical development of number, operations, number systems, and quantity.
NCTM Indicator C.1.5 Historical perspectives of number, operations, number systems, and quantity.	Essay describes the historical perspectives of number, operations, number systems, and quantity in depth and provides specific examples.	Essay describes the historical perspectives of number, operations, number systems, and quantity and provides specific examples.	Essay describes the historical perspectives of number, operations, number systems, and quantity and provides an example.	Essay includes incomplete description of historical perspectives of number, operations, number systems, and quantity.
NCTM Indicator C.1.5 Contributions of historically significant figures and diverse cultures.	Essay describes contributions of historically significant figures and diverse cultures in depth and provides specific examples.	Essay describes contributions of historically significant figures and diverse cultures and provides specific examples.	Essay describes contributions of historically significant figures and diverse cultures and provides an example.	Essay includes incomplete description of historically significant figures and diverse cultures.