George Mason University
College of Education and Human Development
Mathematics Education Leadership

MATH 614-6M1 Rational Numbers and Proportional Reasoning for K-8 Teachers
3 Credits, Spring 2017
Wednesdays, Jan 25 – May 17, 4:30 – 7:10
James Wood Middle School Rm. 106 – Frederick County Public Schools

Faculty

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Prerequisites/Co-requisites

Admission to the Mathematics Education Leadership Master's Degree Program or instructor permission

University Catalog Course Description

This course will cover the basic number strands in fractions and rational numbers, decimals and percents, and ratios and proportions in the school curriculum. Instruction will cover interpretations, computations, and estimation with a coordinated program of activities that develop both rational number concepts and skills and proportional reasoning.

Course Overview

The course, for future K-8 mathematics specialists, examines concepts contained in the number and operations strands of the Virginia Standards of Learning (SOL), Common Core State Standards (CCSS), and the National Council of Teachers of Mathematics (NCTM) Principles and Standards. It is designed to develop a comprehensive understanding of rational numbers and how the structure of this system is related to computations and problem solving. Special attention will be given to children's thinking, how they learn this basic mathematics, their problem solving strategies, and how they construct their understanding of rational numbers and proportional reasoning.

Course Delivery Method

This course will be delivered using a lab format. Class sessions will engage participants in hands-on, in-depth inquiry of mathematics. Participants will need access to the Internet and the GMU Blackboard System: http://mymason.gmu.edu

Learner Outcomes

This course is designed to enable students to do the following:

- Use numerous representations and conceptual models
- Develop flexibility in problem solving
- Explain rational number concepts and interpret student work in many ways

NCTM CAEP Mathematics Content for Elementary Mathematics Specialist

C.1.1 Counting and cardinality, comparing and ordering, understanding the structure of the base ten number system with particular attention to place value, order of magnitude, one-to-one correspondence, properties, and relationships in numbers and number systems – rationals, irrationals, and reals.

C.1.2 Arithmetic operations (addition, subtraction, multiplication, and division) including mental mathematics and standard and non-standard algorithms, interpretations, and representations of numbers – fractions, decimals, rationals, irrationals, and reals.

C.1.4 Quantitative reasoning and relationships that include ratio, rate, proportion, and the use of units in problem situations.

Required Texts

Empson, S. B. & Levi, L. (2011). *Extending children's mathematics: Fractions and decimals. Portsmouth*, NH: Heinemann. ISBN 978-0-325-03053-1

Lamon, S. J. (2012). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers* (3rd ed.). New York: Routledge. ISBN 978-0-415-88612-3

Progressions for the Common Core State Standards in Mathematics (draft) – 6-7, Ratios and Proportional Relationships (available free online)

Progressions for the Common Core State Standards in Mathematics (draft) – 3-5, Number and Operations – Fractions, 3-5 (available free online)

Optional Professional Resources

Barnett-Clark, C., Fisher, W., Marks, R., & Ross, S. (2010). *Developing essential understanding of rational numbers for teaching mathematics in grades 3-5*. Reston, VA: National Council of Teachers of Mathematics. ISBN 978-0-87353-630-1.

Chval, K., Lannin, J., Jones, D., Dougherty, B. (2013). *Putting Essential Understanding of Fractions into Practice in Grades 3-5*. Reston, VA: National Council of Teachers of Mathematics. ISBN 978-0-87353-732-2.

Fosnot, C. T.. & Dolk, M. (2002). *Young mathematicians at work: Constructing fractions, decimals, and percents*. Portsmouth, NH: Heinemann. ISBN 978-0-325-00355-9

Lobato, J. E., & Ellis, A. B. (2010). *Developing essential understanding of ratios, proportions, and proportional reasoning for teaching mathematics in grades 6-8*. Reston, VA: National Council of Teachers of Mathematics. ISBN 978-0-87353-622-6.

Van de Walle, J., Karp, K, & Bay-Williams, J. (2012). *Elementary and middle school mathematics: Teaching developmentally* (7th or 8th edition). Boston, MA: Pearson Education.

Course Performance Evaluation

Students are expected to submit all assignments on time in the manner outlined by the instructor (e.g., Blackboard, Tk20, hard copy). If you are absent from class, assignments due on the day of the missed class are still due and will be considered late if they are not turned in on or before that day. Late assignments will be accepted. However, 2% of the assignment points will be deducted from the assignment grade for each calendar day the assignment is late.

- **Assignments and/or Examinations**

Problem Sets and Class Participation: Class activities and discussions are based upon assigned readings and mathematics problem sets. Readings and problem sets are to be completed before each class session. Students are expected to analyze and reflect upon the readings and come to class prepared to discuss them. Work on problem sets will be discussed in class, and students should be prepared to share their mathematical problem-solving strategies. Problem sets may be collected and evaluated on occasion.

Content Reflection Papers: Students will write three reflections on content studied during the

course. This is a Performance-Based Assessment that is turned in via TK20. Reflections will be graded using the NCTM CAEP Standards Rubric (see below) and will incorporate evidence of student's mathematics content knowledge and mathematical practices shown during class discussions up to that point in the course. Written reflections should include three major parts: 1) a description of the mathematics content (i.e., a mathematical problem or strategy) and example of the students' thinking about that content; 2) a description of mathematical practices used to understand the mathematics content; and 3) a reflection on changes in the students' own understanding and thinking with regard to the content and practices described. Reflections should be 5-6 pages of double-spaced, 12 point font with 1 inch margins. Digital hand-drawn mathematical representations and/or solution strategies may be included. (See Assignment Guidelines for more specific information.)

Student Interview and Analysis Project: Students will conduct two assessment interviews with one elementary-age child and analyze the child's mathematical thinking. This project has five parts: 1) design a plan for the initial interview that includes equal sharing and multiple groups problems, 2) conduct the initial interview, 3) analyze the initial interview and design a plan for the second interview, 4) conduct the second interview, and 5) write a report describing the child's mathematical understandings based upon evidence from the two interviews. (See Assignment Guidelines for more specific information.)

Exams: Students will take a midterm and a final exam covering the content studied in the course. The main focus of the exams will be on the mathematical content of the course. Students will be expected to demonstrate their own understanding and reasoning of the content as well as the knowledge and understanding needed by K-8 students in order to make sense of this content.

- **Other Requirements**
  This course is highly interactive – dependent on both discussions and activities to meet the objectives contained in the course description. Successful completion of the course requires regular attendance, participation, and punctual completion of all class activities, homework, and other assignments. Students must notify the course instructor in advance if they will be absent from class.

- **TK20/Performance-Based Assessment(s) Submission Requirement**
  Every student registered for any Mathematics Education Leadership course with a required TK20 performance-based assessment must submit these assessments (MATH 614 Content Reflections) to TK20 through 'Assessments' in Blackboard. Failure to submit the assessments to TK20 (through Blackboard) will result in the course instructor reporting the course grade as Incomplete (IN). Unless this grade is changed upon completion of the required TK20 submission, the IN will convert to an F nine weeks into the following semester.

- **Grading**

  Grades will be assigned using the following percentages:

Problem sets and class participation	10%
3 Content Reflection Papers (10% each)	30%
Student Interview and Analysis Project	20%
Midterm Exam	20%
Final Exam	20%

  The course will be graded on a 100 point system:

  100-98 A+ 97-90 A 89-87 B+ 86-80 B 79-70 C 69 and lower F

- **NCTM CAEP Standards Rubric**
  Evidence of each criteria listed below will be indicated using the following rubric:

No Evidence 1	Minimal Evidence 2	Adequate Evidence 3	Exceptional Evidence 4
The candidate did not show evidence of the criteria.	The candidate shows minimal evidence of the criteria.	The candidate shows adequate evidence of the criteria.	The candidate shows exceptional evidence of the criteria.

Standard 1: Content Knowledge

Element	Criteria	
1.a.1	Demonstrate knowledge of major concepts, algorithms, and procedures within and among mathematical domains.	
1.a.2	Apply knowledge of major concepts, algorithms, procedures, applications in varied contexts, and connections within and among mathematical domains.	
1.a.3	Explain how concepts, algorithms, procedures, and applications have developed.	
1.a.4	Apply conceptual and procedural knowledge of major concepts, algorithms, and applications in building new knowledge from prior knowledge and experiences.	

Standard 2: Mathematical Practices

Element	Criteria	
2.a.1	Use problem solving to develop conceptual understanding and to formulate and test generalizations.	
2.a.2	Make sense of a wide variety of problems and persevere in solving them.	
2.a.3	Apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts.	
2.a.4	Formulate and test conjectures in order to frame generalizations.	
2.a.5	Monitor and reflect on the process of mathematical problem solving.	
2.b.1	Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others.	
2.b.2	Represent and model generalizations using mathematics.	
2.b.3	Recognize structure and express regularity in patterns of mathematical reasoning.	
2.b.4	Use multiple representations to model and describe mathematics.	
2.b.5	Use appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.	
2.b.6	Demonstrate an appreciation for mathematical rigor and inquiry.	
2.c.1	Formulate, represent, analyze, interpret, and validate mathematical models derived from real-world contexts or mathematical problems.	
2.c.2	Demonstrate flexibility in mathematical modeling when confronted with different purposes or contexts.	
2.f.1	Model how the development of mathematical understanding within and among mathematical domains intersects with the mathematical practices of problem solving, reasoning, communicating, connecting, and representing.	

2.f.2	Reflect on how the mathematical practices of problem solving, reasoning, communicating, connecting, and representing impact mathematical understanding.	

Class Schedule

Session	Topic/Learning Experience	Readings & Assignments Due on this Date
1 1/25/17	▪ Course Introduction	▪ Selected problems from Lamon and Empson (completed in class)
2 2/01/17	▪ Overview of proportional reasoning ▪ Invariance and covariance ▪ Multiplicative thinking ▪ Equal sharing problems and children's strategies for solving them	▪ Lamon: Chapter 1 (selected problems) ▪ Empson & Levi: Foreword, Introduction, Chapter 1, & Chapter 2 (selected problems)
3 2/08/17	▪ Sets of numbers ▪ Definitions of fractions and rational numbers ▪ Multiple interpretations ▪ Multiple groups problems and children's strategies for solving them	▪ Lamon: Chapter 2 (selected problems) ▪ Empson & Levi: Chapter 3 (selected problems)
4 2/15/17	▪ Relative thinking ▪ Measurement ▪ Relational thinking: Connecting fractions and algebra	▪ Lamon: Chapter 3 (selected problems) ▪ Empson & Levi: Chapters 4 & 5
5 2/22/17	▪ Quantities and covariation ▪ Proportional relationships ▪ Scale Factors ▪ Similarity ▪ Fraction equivalence and order	▪ Lamon: Chapter 4 (selected problems) ▪ **Content Reflection #1 due**
6 3/01/17	▪ Proportional reasoning ▪ Reasoning up and down ▪ Units and Unitizing ▪ Ratio tables	▪ Lamon: Chapter 5 (selected problems)
7 3/08/17	▪ Reasoning with fractions ▪ Equivalent fractions ▪ Comparing and ordering fractions	▪ Lamon: Chapter 6 (selected problems) ▪ Empson & Levi: Chapter 6 (selected problems) ▪ **Student Interview Check Point – Turn in your plan for review**
8 3/15/17	**Midterm Exam**	▪ Midterm Review Problem Set ▪ **Content Reflection #2 due**
3/22/17	▪ Fractions as part-whole comparisons ▪ Fractions models	▪ Lamon: Chapter 7 (selected problems) ▪ Empson & Levi: Chapter 7

	▪ Multiplication and division of fractions ▪ Understanding decimals	(selected problems)
9 3/29/17	▪ Fractions as quotients ▪ Partitioning ▪ Understanding operations on fractions and decimals	▪ Lamon: Chapter 8 (selected problems) ▪ Empson & Levi: Chapter 8 (selected problems) ▪ **Student Interview #1 due**
10 4/5/17	▪ Fractions as operators ▪ Exchange models ▪ Area model for multiplication and division	▪ Lamon: Chapter 9 (selected problems)
11 4/12/17	▪ Fractions as measures ▪ Successive partitioning	▪ Lamon: Chapter 10 (selected problems) ▪ **Student Interview #2 due**
12 4/19/17	SPRING BREAK	
13 4/26/17	▪ Ratios and Rates ▪ Equivalence and comparison of ratios	▪ Lamon: Chapter 11 (selected problems) ▪ **Student Interview Report due**
14 5/03/17	▪ Changing instruction ▪ Learning to use children's thinking to guide instruction	▪ Lamon: Chapter 12 ▪ Empson: Chapter 9 ▪ **Content Reflection #3 due**
5/10/17	READING DAY No Class	
15 5/17/17	**Final Exam**	● Final Review Problem Set

Note: Faculty reserves the right to alter the schedule as necessary, with notification to students.

Professional Dispositions

Students are expected to exhibit professional behaviors and dispositions at all times.

Core Values Commitment

The College of Education and Human Development is committed to collaboration, ethical leadership, innovation, research-based practice, and social justice. Students are expected to adhere to these principles: http://cehd.gmu.edu/values/.

GMU Policies and Resources for Students

Policies

- Students must adhere to the guidelines of the Mason Honor Code (see http://oai.gmu.edu/the-mason-honor-code/).

- Students must follow the university policy for Responsible Use of Computing (see http://universitypolicy.gmu.edu/policies/responsible-use-of-computing/).

- Students are responsible for the content of university communications sent to their Mason email account and are required to activate their account and check it regularly. All communication from the university, college, school, and program will be sent to students **solely** through their Mason email account.

- Students with disabilities who seek accommodations in a course must be registered with George Mason University Disability Services. Approved accommodations will begin at the time the written letter from Disability Services is received by the instructor (see http://ods.gmu.edu/).

- Students must follow the university policy stating that all sound emitting devices shall be silenced during class unless otherwise authorized by the instructor.

Campus Resources

- Support for submission of assignments to Tk20 should be directed to tk20help@gmu.edu or https://cehd.gmu.edu/aero/tk20. Questions or concerns regarding use of Blackboard should be directed to http://coursessupport.gmu.edu/.

- The Writing Center provides a variety of resources and services (e.g., tutoring, workshops, writing guides, handbooks) intended to support students as they work to construct and share knowledge through writing (see http://writingcenter.gmu.edu/).

- The Counseling and Psychological Services (CAPS) staff consists of professional counseling and clinical psychologists, social workers, and counselors who offer a wide range of services (e.g., individual and group counseling, workshops and outreach programs) to enhance students' personal experience and academic performance (see http://caps.gmu.edu/).

- The Student Support & Advocacy Center staff helps students develop and maintain healthy lifestyles through confidential one-on-one support as well as through interactive programs and resources. Some of the topics they address are healthy relationships, stress

management, nutrition, sexual assault, drug and alcohol use, and sexual health (see http://ssac.gmu.edu/). Students in need of these services may contact the office by phone at 703-993-3686. Concerned students, faculty and staff may also make a referral to express concern for the safety or well-being of a Mason student or the community by going to http://ssac.gmu.edu/make-a-referral/.

For additional information on the College of Education and Human Development, please visit our website https://cehd.gmu.edu/.