

George Mason University
Department of Mathematical Sciences in cooperation with
Graduate School of Education

Special topics: MATH 600 (3 credits)

Rational Number/ Proportional Reasoning and Assessment in the Middle Grades
Summer 2014

Professional Development Outreach Course

Center for Outreach in Mathematics Professional Learning and Educational Technology

Course Organizers and Instructors:

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I. Course Description:

Assessing through Problem-based Tasks and Unpacking the Mathematical Learning Progressions in grades 6-8

This course focuses on mathematical inquiry through understanding how student learning progresses in the domains of Rational Numbers, Proportional Reasoning, Functions and Algebra, Data Analysis & Probability, Measurement and Geometry. This class enhances middle school teacher content knowledge of rational numbers, ratios and proportional reasoning through (a) Quantitative Proportional Reasoning (QPR), (b) Algebraic Proportional Reasoning (APR), and (c) Spatial Proportional Reasoning (SPR). This course covers Virginia SOL strands in fractions, ratios and rational numbers. Instruction covers interpretations, computation, and estimation with fractions, ratios, proportions, decimals, and percents through a coordinated program of activities that develop rational number concepts and skills. This course will engage participants in a coordinated program that includes hands-on activities and in-depth discussions that develop both rational number concepts and proportional reasoning. Attention will be given to 6 – 8 students' development and understanding of fractions, ratios, proportions, decimals and percents, and ultimately rational numbers and proportional reasoning. Special attention will be given to interpreting and assessing students' work and learning.

Course Objectives: Participants will

- Increase professional competence, confidence and enthusiasm for teaching and learning mathematics.
- Deepen their understanding of rational numbers and proportional reasoning including the development of a variety of strategies for thinking about and working with concepts in these areas and will be able to use multiple representations of rational numbers and conceptual models to demonstrate flexibility in problem solving.
- Deepen their understanding of how children develop rational number sense and learn rational numbers and proportional reasoning in grades 6 – 8 as is discussed in NCTM reform documents.

- Examine young adolescent's mathematical thinking through student work and plan opportunities for further learning.
- Examine middle grades curricular materials and develop planning strategies.
- Examine Lesson Study as a collaborative learning/planning model for teachers and participate in a modified lesson study cycle
- Identify and develop appropriate teaching strategies. Attention will be given to selecting appropriate and worthwhile tasks, asking probing questions, guiding conversation and selecting ideas to be shared in an effort to meet each "horizon," and evaluating student thinking/work.
- Examine research related to the learning and teaching of mathematics in the middle grades and become knowledgeable about current recommendations and trends in the mathematics education community regarding content in school mathematics.
- Take responsibility for the class community's professional growth and contribute to the field of mathematics education, specifically in the state of Virginia.

Class Meetings: The meeting dates are as follows:

- Summer Institute: July 28 - Aug 1 from 8:30-3:00pm, **GMU Prince William Campus**, 10900 University Blvd., Manassas, VA
- Face-to-face meeting: September 27, 2014 Location: TBD
- follow-up webinar meeting **Date: TBD**

II. Student Outcomes

Participants will investigate and develop fluency in multiple interpretations of rational numbers and be able to use these interpretations to flexibly solve complex problems involving rates, ratios, and proportional reasoning. Specific content of this course will include:

- The role and importance of context
- The idea of equivalence; especially between fractions
- The historical roots of equivalence, early numbers systems and equivalent fractions
- Extending equivalence to include percents and decimals
- Multiplication and division of fractions
- Developing mathematical models and their use in generalizations
- Developing computational strategies, leading to efficient computational strategies
- The multiple meanings and interpretations of rational numbers
- The importance of identifying units (wholes) in fraction problems
- Rational numbers as measures and operators; partitioning and quotients; and ratios, rates and proportions

At the conclusion of this course, students should be able to:

- Promote a better understanding of the nature of mathematics, learning progressions and mathematical inquiry
- Demonstrate problem-solving strategies in various mathematical content areas and methods for cultivating problems solving, reasoning and communicating skills
- Foster an understanding of how children's mathematical thinking develops

- Articulate methodologies for teaching mathematics more effectively to children with various abilities in Grades 6-8; Plan effective mathematics instruction for students from diverse populations with a variety of learning needs

III. Nature of Course Delivery

The delivery of this course combines methods of seminar, online sessions, active learning, discussion, independent work, student presentation, mathematical problem solving, and writing. The course is designed both in structure and process to engage students in dialogue at the individual, group, and collective levels. Different formats will be used to help build both the capacity of the learning community. Readings and lectures will precede and focus class on-line discussion and interactive forums. This course relies on your willingness to participate in all class and team discussions. You will be asked to complete reading assignments and offer key ideas on how the readings inform professional experience. The syllabus provides an initial plan for our work and may be revised during the course to meet students' needs and interests. Students are expected to be independent thinkers, intellectually curious, and responsible to each other for the quality of classroom learning. This calls for both purposeful collaborative work as well as deep individual reflection. The course is designed to enhance both of these skill sets. You should expect to spend time in between classes to reading/viewing/listening to assigned materials, conducting research and completing assignments, completing reflections, problem solving and simulations, and participating in substantive on-line discussions.

IV. Readings: Reading packet & Resources on Class website

Lamon, S.J. (2012). *Teaching fractions and ratios for understanding (3rd ed)*. Taylor and Francis

Burrill, J. (Ed.). (2005). *Mathematics assessment sampler, grades 6-8: items aligned with NCTM's principles and standards for school mathematics*. Reston, VA: NCTM.

V. Course Requirements and Assignments

The assignments across the semester are intended to improve your strategies as a mathematics teacher and to develop your skills in the interpretation, critique and synthesis of mathematics education research. All assignments are to be completed on time so that class members might benefit from the expertise and contributions of their colleagues.

A. Participation, Postings and Reflections (30%)

Class Participation: Class seminars will consist of a discussion of the readings and related problems. Readings and assigned problems are to be completed before each class seminars. Students are expected to analyze and reflect on the readings and problem solving strategies and come to class prepared to participate in the discussion.

Posting and Reflections: Participants will write reflections in order to process mathematical ideas, mathematical learning progressions, and pedagogy that are discussed in the seminars and highlighted in the readings.

B. Cognitively Demanding Task Group Presentation (40%)

In this course we will study the teaching and learning of ideas related to rational number and proportional reasoning in 6-8 mathematics and how to encourage a productive disposition in both our students and ourselves. As a summative evaluation of the pedagogical aspects of this course, your group will develop or select a task which fosters learning these concepts, teach and document the task, and give a short (15-minute) presentation to the class on it. The lesson draft checkpoint includes items a-e below. The final lesson paper your group submits must include all of the following components:

- a) Select or develop a cognitively demanding task intended for use with the students you teach, which encourages student development of rational number and proportional reasoning. You may use or adapt a problem from class materials, but be sure it is appropriate for the target audience. Specify prerequisite knowledge. Solve the problem, then as a group compile a collection of possible solution strategies.
- b) Write a paragraph explaining what essential rational number/ proportional understanding is addressed in this problem and your specific learning goal for the lesson. Include a summary of the problem solving strategies you anticipate seeing in the classroom as you teach this lesson. Identify which strategies you will select for use in your classroom.
Present the strategies in the sequence that your group has determined including the sequence rationale. And lastly, explain how the strategies connect to one another, to prior knowledge or to future learning.
- c) Write a lesson plan using the provided template that uses the problem/activity as a significant problem-solving opportunity for your students. Your group may all use the same lesson plan and include an appendix to specify any class/grade specific changes.
- d) Each member of the group will teach the lesson to his/her students and write a one-page reflection on how the lesson went, including what strategies students used to approach the problem, what ideas were raised in its discussion, and to what extent your students' understanding of the underlying rational number/ proportional reasoning concepts or ability to apply them changed as a result of the lesson. Be specific. Synthesize the individual reflections into a single reflection that will include what went as planned, what was surprising and what changes would be made to the original lesson.
- e) Create a one-page handout (you may use front and back if necessary, but it must fit on one sheet) summarizing your lesson for the class. Include the problem, grade level(s), mathematical topics addressed, and anything your colleagues would need to know in order to use the lesson, including (briefly) any difficulties the students tended to encounter. The handout should not be the same as your lesson plan (just select details!), and will be turned in with the main paper. On the day of your group's presentation bring a copy for each class member.

C. Student Thinking Analysis (20%)

During the course we will read and discuss student thinking and pedagogical strategies found in the classroom related to proportional reasoning. For this assignment, you are to write a short (roughly 3-5 page) case study describing a mathematical discussion involving one or more students, as they tackle a proportional reasoning problem. A case is neither a complete

transcript of a lesson nor as prefabricated as an interview, although it is very helpful to include direct quotes and dialogue from students. You must base your case on a conversation for which you were present, and preferably in which you were involved, but it could come out of a lesson you observed, or a conversation among two or more students. You may choose to narrow in on one or two students, or on one small group, or you may describe a whole-class conversation. The most important thing is that the episode illustrates some aspect of children's mathematical thinking about proportionality and centers on a mathematical topic involving children's thinking about rational number and proportional reasoning.

In writing your case study, begin by describing briefly the class's larger context (including grade level) and the mathematical topic; then describe the relevant parts of the conversation in as much detail as you can manage. Include what you are thinking as you work with the students. Finish up by summarizing your evaluation of the students involved and saying what issues and questions you still have after this conversation. Include an analysis of the students' thinking, and questions the case raises for you. It is important that your reflection address teaching issues beyond the one topic and set of students involved. This will document your ability as a reflective practitioner to make connections that inform your teaching practice more broadly.

D. Final Content Exam (10%)

Participants will take comprehensive exam covering the content studied in the course. The main focus of the exam will be on the mathematical content of the course. Students will be expected to demonstrate their own understanding and reasoning of the content as well as the knowledge and understanding needed by K-5 students in order to make sense of this content.

VI. Evaluation Schema

Determination of the Final Grade:

Graduate Grading Scale

A 93%-100%

B+ 87%-89%

C 70%-79%

A- 90%-92%

B 80%-86%

F Below 70%

VII. UNIVERSITY POLICIES

The university has a policy that requests students to turn off pagers and cell phones before class begins.

Formative Assessment:

<http://www.smarterbalanced.org/sample-items-and-performance-tasks/>

<http://www.parcconline.org/K2-assessments>

http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html

<http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/misunderstandings.aspx>

<http://map.mathshell.org/materials/index.php>

<http://www.exemplars.com/resources/formative/index.html>

<http://mathforum.org/mathed/assessment.html>

<http://balancedassessment.concord.org/>

Explorelarning(Gizmo):www.explorelarning.com

AIMS<http://www.aimsedu.org/>

Middle school Contextualized Problems: <http://www.mmmproject.org/data.htm>

Model Eliciting Tasks: <http://crlt.indiana.edu/research/csk.html>

HONOR CODE

To promote a stronger sense of mutual responsibility, respect, trust, and fairness among all members of George Mason University and with the desire for greater academic and personal achievement, George Mason University has set forth a code of honor that includes policies on cheating and attempted cheating, plagiarism, lying and stealing. Detailed information on these policies is available in the GMU Student Handbook, the University Catalog, and on the GMU website (www.gmu.edu).

Individuals with Disabilities Policy

The university is committed to complying with the Rehabilitation Act of 1973 and the Americans with Disabilities Act of 1990 by providing reasonable accommodations for applicants for admission, students, applicants for employment, employees, and visitors who are disabled. Applicants for admission and students requiring specific accommodations for a disability should contact the Disability Resource Center at 703-993-2474, or the University Equity Office at 703-993-8730.

ATTENDANCE POLICY

Students are expected to attend the class periods of the courses for which they register. Although absence alone is not a reason for lowering a grade, students are not relieved of the obligation to fulfill course assignments, including those that can only be fulfilled in class. Students who fail to participate (because of absences) in a course in which participation is a factor in evaluation, or students who miss an exam without an excuse, may be penalized according to the weighted value of the missed work as stated in the course syllabus (GMU University Catalog, pg. 32).

TASKSTREAM REQUIREMENTS

Every student registered for any MEL course with a required performance-based assessment (will be designated as such in the syllabus) is required to submit this assessment (*Professional Development Grant Proposal*) to TaskStream (regardless of whether a course is an elective, a onetime course or part of an undergraduate minor). Evaluation of your performance-based assessment will also be provided using TaskStream. Failure to submit the assessment to TaskStream will result in a the course instructor reporting the course grade as Incomplete(IN). Unless this grade is changed upon completion of the required TaskStream submission, the IN will convert to an F nine weeks into the following semester.

GMU POLICIES AND RESOURCES FOR STUDENTS

- a. Students must adhere to the guidelines of the George Mason University Honor Code [See <http://oai.gmu.edu/honor-code/>].
- b. Students must follow the university policy for Responsible Use of Computing [See <http://universitypolicy.gmu.edu/1301gen.html>].
- c. Students are responsible for the content of university communications sent to their George Mason University email account and are required to activate their account and check it regularly. All communication from the university, college, school, and program will be sent to students solely through their Mason email account.
- d. The George Mason University Counseling and Psychological Services (CAPS) staff consists of professional counseling and clinical psychologists, social workers, and counselors who offer a wide range of services (e.g., individual and group counseling, workshops and outreach programs) to enhance students' personal experience and academic performance [See <http://caps.gmu.edu/>].
- e. Students with disabilities who seek accommodations in a course must be registered with the George Mason University Office of Disability Services (ODS) and inform their instructor, in writing, at the beginning of the semester [See <http://ods.gmu.edu/>].
- f. Students must follow the university policy stating that all sound emitting devices shall be turned off during class unless otherwise authorized by the instructor.
- g. The George Mason University Writing Center staff provides a variety of resources and services (e.g., tutoring, workshops, writing guides, handbooks) intended to support students as they work to construct and share knowledge through writing [See <http://writingcenter.gmu.edu/>].

PROFESSIONAL DISPOSITIONS

Students are expected to exhibit professional behaviors and dispositions at all times.

CORE VALUES COMMITMENT

The College of Education & Human Development is committed to collaboration, ethical leadership, innovation, research-based practice, and social justice. Students are expected to adhere to these principles. <http://cehd.gmu.edu/values/>

For additional information on the College of Education and Human Development, Graduate School of Education, please visit our website [See <http://gse.gmu.edu/>].