Quantitative Research in Education
Intermediate & Advanced Methods

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Whittier Publications, Inc.
New York
PREFACE

This book offers a comprehensive presentation of quantitative research design and statistical methods in the context of education and related fields. The text is intended primarily for use by students who take intermediate and advanced quantitative research courses as a part of their graduate degree program, but it can be a useful resource for researchers in education, counseling, rehabilitation, psychology, sociology, social work, and human development as well.

The main purpose of this book is to provide the readers with an in-depth conceptual and methodological understanding of intermediate and advanced quantitative research methods, as well as the skills necessary to apply such methods using SPSS and to interpret the results. This is achieved by building layers of context-based understanding of research concepts and methods, their statistical translation, methodological principles, computer-based data analysis, presentation of the results in APA style format, and contextual interpretations. The text allows people who experience difficulties with analytic representations of statistical concepts to capitalize on conceptual understanding and still be able to master the research tools necessary for their work on theses, dissertations, and professional research.

While there are many excellent introductory books on research design and statistics in education and the social sciences, most books at the intermediate and advanced levels tend to be either too technical and mathematical or too simplistic. Typically, claiming to have an "applied orientation," such books are dominated by presentations of SPSS dialog boxes and printouts at the expense of theoretical and methodological rigor. To bridge the gap between these extremes, this book attempts to provide a balance between conceptual meaning and its statistical translation by developing understanding and application skills in a spiral exposure to quantitative concepts and methods. For example, the comparison of groups on variables of interest is addressed in a sequence from univariate cases of $t$-tests, nonparametric methods, and analysis of variance (ANOVA) to scenarios illustrating the use of multivariate analysis of variance (MANOVA) and structural equation modeling (SEM). As another example, the concept of validity is addressed in the framework of measurement, research design, and structural equation modeling. Particular attention is devoted to potential problems associated with violation of assumptions, common misconceptions (e.g., conducting MANOVA versus separate ANOVAs), effect sizes, confidence intervals, and sample size. The book is organized in four parts comprising 24 chapters. Each chapter ends with a summary and study questions.

**Part I [Measurement in Educational Research]** consists of three chapters. Chapter 1 presents variables and measurement scales in the context of education. The focus is on the nature of measurement in education, types of variables, types of scales and their transformations, permissible arithmetic operations with scale values, summation symbols, and basic rules of summation. Chapter 2 introduces the classical model of reliability of scores, types of reliability, and reliability of composite scores. Chapter 3 deals with the concept of validity for measurement instruments (e.g., tests, questionnaires, or inventories) and types of validity (content-related validity, criterion-related validity, and construct-related validity).

**Part II [Research Design]** consists of two chapters. Chapter 4 deals with research problems, hypotheses, and types of quantitative research: nonexperimental research, experimental research, and threats to internal and external validity. Chapter 5 presents pre-experimental and true experimental research designs that involve quantitative methods of data analysis. The focus is primarily on conceptual understanding and methodological principles underlying the application of such designs in educational research.
Part III [Univariate Statistics in Educational Research] consists of fourteen chapters. The first five of these chapters (6, 7, 8, 9, and 10) cover introductory statistics and prepare the ground for understanding and practical applications of intermediate statistics in educational research. The next six chapters (11 through 16) provide intermediate treatment of correlation, regression, and analysis of variance (ANOVA) including some nonparametric methods. The last three chapters in this section (17, 18, and 19) provide more advanced treatment of multiple regression, analysis of variance, and the relations between them.

Part IV [Multivariate Statistics in Educational Research] consists of five chapters. This part covers the topics of logistic regression, multivariate analysis of variance (MANOVA), exploratory factor analysis, confirmatory factor analysis, and elements of structural equation modeling (SEM). The analytic framework of these topics is simplified and tailored to conceptual understanding, computer-aided applications, and interpretations in the context of educational research.

Supplements
Data sets for computer-based applications in examples using SPSS can be downloaded from the online supplement to this book [http://cehd.gmu.edu/book/dimitrov]. This supplement provides also (a) answers to the study questions for each chapter, (b) addendum to some topics discussed in the book, (c) syntax for confirmatory factor analysis, path analysis, and group comparison on latent variables in the framework of major computer programs — LISREL, AMOS, EQS, and Mplus [used for illustrations in Chapters 23 and 24], and (d) additional references (books, articles, and online products) related to the content of this book.

Acknowledgments
I would like to acknowledge the assistance of my graduate research assistant, Jill Lammert, who provided valuable editorial comments and suggestions throughout the writing of this book. I also appreciate the feedback and encouragement of colleagues from different universities as well as graduate students who used draft chapters of this book in quantitative research courses that they took with me in the Graduate School of Education at George Mason University.

Dimiter M. Dimitrov
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